JEE Main 2021

24 FEBRUARY SHIFT I

PHYSICS

Section A : Objective Type Questions

1. The work done by a gas molecule in an isolated system is given by, $W = \alpha\beta^2 e^{-\frac{x^2}{\alpha kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature, α and β are constants.

Then, the dimensions of β will be

a. $[M^2 L T^2]$ **b.** $[M^0 L T^0]$ **c.** $[M L T^{-2}]$ **d.** $[M L^2 T^{-2}]$

2. Two stars of masses m and $2m$ at a distance d rotate about their common centre of mass in free space. The period of revolution is

a. $\frac{1}{2\pi}\sqrt{\frac{3Gm}{d^3}}$ **b.** $2\pi\sqrt{\frac{d^3}{3Gm}}$

c. $2\pi\sqrt{\frac{3Gm}{d^3}}$ **d.** $\frac{1}{2\pi}\sqrt{\frac{d^3}{3Gm}}$

3. Four identical particles of equal masses 1 kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be

a. $\sqrt{\frac{(1+2\sqrt{2})G}{2}}$ **b.** $\sqrt{\frac{G}{2}(1+2\sqrt{2})}$

c. $\sqrt{G(1+2\sqrt{2})}$ **d.** $\sqrt{\frac{G}{2}(2\sqrt{2}-1)}$

4. Moment of inertia (MI) of four bodies, having same mass and radius, are reported as

I_1 = MI of thin circular ring about its diameter,

I_2 = MI of circular disk about an axis perpendicular to the disk and going through the centre,

I_3 = MI of solid cylinder about its axis

and I_4 = MI of solid sphere about its diameter. Then,

a. $I_1 + I_2 = I_3 + \frac{5}{2}I_4$ **b.** $I_1 + I_3 < I_2 + I_4$

c. $I_1 = I_2 = I_3 < I_4$ **d.** $I_1 = I_2 = I_3 > I_4$

5. Consider two satellites S_1 and S_2 with periods of revolution 1 h and 8 h respectively, revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellite S_2 is

a. 8 : 1 **b.** 1 : 8 **c.** 2 : 1 **d.** 1 : 4

6. Each side of a box made of metal sheet in cubic shape is a at room temperature T, the coefficient of linear expansion of the metal sheet is α. The metal sheet is heated uniformly, by a small temperature ΔT, so that its new temperature is $T + \Delta T$. Calculate the increase in the volume of the metal box.

a. $4\pi a^3\alpha\Delta T$ **b.** $4a^3\alpha\Delta T$

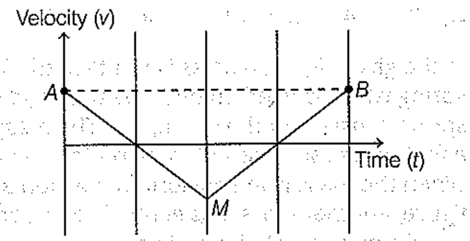
c. $\frac{4}{3}\pi a^3\alpha\Delta T$ **d.** $3a^3\alpha\Delta T$

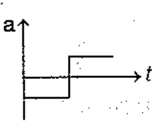
7. If Y, K and η are the values of Young's modulus, bulk modulus and modulus of rigidity of any material, respectively. Choose the correct relation for these parameters.

a. $Y = \frac{9K\eta}{2\eta + 3K}$ N/m^2 **b.** $Y = \frac{9K\eta}{3K - \eta}$ N/m^2

c. $K = \frac{Y\eta}{9\eta - 3Y}$ N/m^2 **d.** $\eta = \frac{3YK}{9K + Y}$ N/m^2

8. If the velocity-time graph has the shape AMB, what would be the shape of the corresponding acceleration-time graph ?



a.


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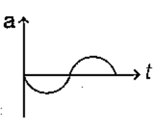

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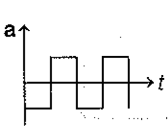

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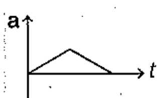


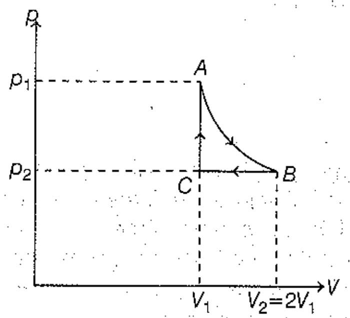
9. n mole of a perfect gas undergoes a cyclic process $ABCA$ (see figure) consisting of the following processes.

$A \to B$: Isothermal expansion at temperature T, so that the volume is doubled from V_1 to $V_2 = 2V_1$ and pressure changes from p_1 to p_2.

$B \rightarrow C$: Isobaric compression at pressure p_2 to initial volume V_1.

$C \rightarrow A$: Isochoric change leading to change of pressure from p_2 to p_1.

Total work done in the complete cycle $ABCA$ is



a. 0
b. $nRT \ln 2$
c. $nRT\left(\ln 2+\frac{1}{2}\right)$
d. $nRT\left(\ln 2-\frac{1}{2}\right)$

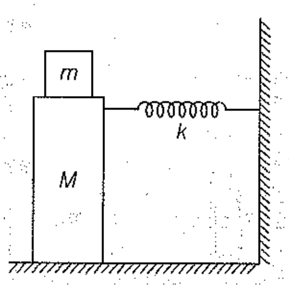
10. Match List-I with List-II.

	List-I		List-II
A.	Isothermal	1.	Pressure constant
B.	Isochoric	2.	Temperature constant
C.	Adiabatic	3.	Volume constant
D.	Isobaric	4.	Heat content is constant

Choose the correct answer from the options given below.

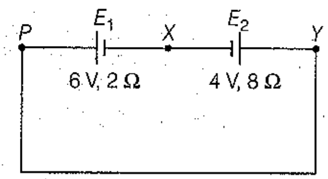
	A	B	C	D		A	B	C	D
a.	1	3	2	4	b.	3	2	1	4
c.	2	4	3	1	d.	2	3	4	1

11. In the given figure, a mass M is attached to a horizontal spring which is fixed on one side to a rigid support. The spring constant of the spring is k. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass is in equilibrium position as shown in the figure, another mass m is gently fixed upon it. The new amplitude of oscillation will be



a. $A\sqrt{\frac{M+m}{M}}$
b. $A\sqrt{\frac{M}{M+m}}$
c. $A\sqrt{\frac{M-m}{M}}$
d. $A\sqrt{\frac{M}{M-m}}$

12. A cell E_1 of emf 6V and internal resistance 2Ω is connected with another cell E_2 of emf 4 V and internal resistance 8Ω (as shown in the figure). The potential difference across points X and Y is



a. 2.0 V
b. 3.6 V
c. 5.6 V
d. 10.0 V

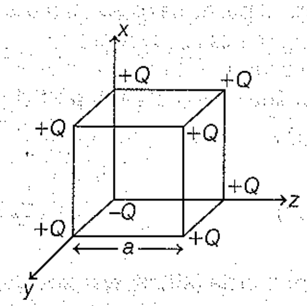
13. A current through a wire depends on time as $i=\alpha_0 t+\beta t^2$, where $\alpha_0 = 20$ A/s and $\beta = 8$ As^{-2}. Find the charge crossed through a section of the wire in 15 s.

a. 260 C
b. 2100 C
c. 11250 C
d. 2250 C

14. Two equal capacitors are first connected in series and then in parallel. The ratio of the equivalent capacities in the two cases will be

a. 1 : 2
b. 2 : 1
c. 4 : 1
d. 1 : 4

15. A cube of side a has point charges $+Q$ located at each of its vertices except at the origin, where the charge is $-Q$. The electric field at the centre of cube is



a. $\frac{-Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}})$

b. $\frac{Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}})$

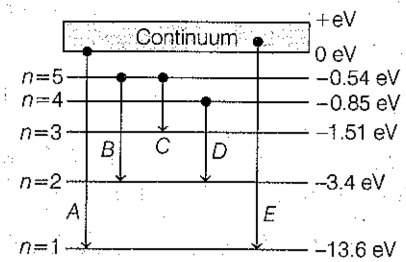
c. $\frac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}})$

d. $\frac{2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}})$

16. If an emitter current is changed by 4 mA, the collector current changes by 3.5 mA. The value of β will be

a. 7
b. 0.875
c. 0.5
d. 3.5

17. In the given figure, the energy levels of hydrogen atom have been shown alongwith some transitions marked *A, B, C, D* and *E*. The transitions *A, B* and *C* respectively represent



a. The first member of the Lyman series, third member of Balmer series and second member of Paschen series.
b. The ionisation potential of hydrogen, second member of Balmer series and third member of Paschen series.
c. The series limit of Lyman series, second member of Balmer series and second member of Paschen series.
d. The series limit of Lyman series, third member of Balmer series and second member of Paschen series.

18. Given below are two statements :

Statement I Two photons having equal linear momenta have equal wavelengths.

Statement II If the wavelength of photon is decreased, then the momentum and energy of a photon will also decrease.

In the light of the above statements, choose the correct answer from the options given below.

a. Both Statement I and Statement II are true.
b. Both Statement I and Statement II are false.
c. Statement I is true but Statement II is false.
d. Statement I is false but Statement II is true.

19. The focal length f is related to the radius of curvature r of the spherical convex mirror by

a. $f = r$ **b.** $f = -r$
c. $f = -\frac{r}{2}$ **d.** $f = +\frac{r}{2}$

20. In a Young's double slit experiment, the width of the one of the slit is three times the other slit. The amplitude of the light coming from a slit is proportional to the slit-width. Find the ratio of the maximum to the minimum intensity in the interference pattern.

a. 4 : 1 **b.** 2 : 1
c. 1 : 4 **d.** 3 : 1

Section B : Numerical Type Questions

21. The coefficient of static friction between a wooden block of mass 0.5 kg and a vertical rough wall is 0.2. The magnitude of horizontal force that should be applied on the block to keep it adhere to the wall will be .... N. [Take, $g = 10\ \text{ms}^{-2}$]

22. An unpolarised light beam is incident on the polariser of a polarisation experiment and the intensity of light beam emerging from the analyser is measured as 100 lumens. Now, if the analyser is rotated around the horizontal axis (direction of light) by 30° in clockwise direction, the intensity of emerging light will be .... lumens.

23. A ball with a speed of 9 m/s collides with another identical ball at rest. After the collision, the direction of each ball makes an angle of 30° with the original direction. The ratio of velocities of the balls after collision is $x : y$, where x is ....

24. A hydraulic press can lift 100 kg when a mass m is placed on the smaller piston. It can lift ...... kg when the diameter of the larger piston is increased by 4 times and that of the smaller piston is decreased by 4 times keeping the same mass m on the smaller piston.

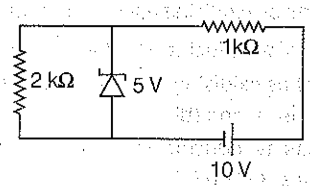
25. An inclined plane is bent in such a way that the vertical cross-section is given by $y = \frac{x^2}{4}$ where, y is in vertical and x in horizontal direction. If the upper surface of this curved plane is rough with coefficient of friction $\mu = 0.5$, the maximum height in cm at which a stationary block will not slip downward is .... cm.

26. A resonance circuit having inductance and resistance 2×10^{-4} H and 6.28 Ω respectively oscillates at 10 MHz frequency. The value of quality factor of this resonator is ....

[Take, $\pi = 3.14$]

27. An audio signal $v_m = 20 \sin 2\pi(1500\,t)$ amplitude modulates a carrier
$v_c = 80 \sin 2\pi(100000\,t)$
The value of per cent modulation is ....

28. In connection with the circuit drawn below, the value of current flowing through 2kΩ resistor is .... $\times 10^{-4}$ A.



29. An electromagnetic wave of frequency 5 GHz, is travelling in a medium whose relative electric permittivity and relative magnetic permeability both are 2. Its velocity in this medium is ...... $\times 10^7$ m/s.

30. A common transistor radio set requires 12 V (DC) for its operation. The DC source is constructed by using a transformer and a rectifier circuit, which are operated at 220 V (AC) on standard domestic AC supply. The number of turns of secondary coil are 24, then the number of turns of primary are ....

CHEMISTRY

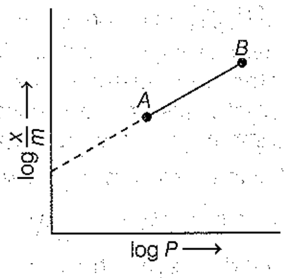
Section A : Objective Type Questions

1. Which of the following are isostructural pairs ?

A. SO_4^{2-} and CrO_4^{2-} B. $SiCl_4$ and $TiCl_4$
C. NH_3 and NO_3^- D. BCl_3 and $BrCl_3$

a. A and B only **b.** A and C only
c. B and C only **d.** C and D only

2. In Freundlich adsorption isotherm, slope of *AB* line is



a. n with ($n = 0.1$ to 0.5) **b.** $\log n$ with ($n > 1$)
c. $\log \frac{1}{n}$ with ($n < 1$) **d.** $\frac{1}{n}$ with $\left(\frac{1}{n} = 0 \text{ to } 1\right)$

3. Consider the elements Mg, Al, S, P and Si, the correct increasing order of their first ionisation enthalpy is

a. Al < Mg < Si < S < P **b.** Mg < Al < Si < P < S
c. Mg < Al < Si < S < P **d.** Al < Mg < S < Si < P

4. Which of the following ore is concentrated using group 1 cyanide salt ?

a. Calamine **b.** Malachite **c.** Siderite **d.** Sphalerite

5. (A) $HOCl + H_2O_2 \longrightarrow H_3O^+ + Cl^- + O_2$

(B) $I_2 + H_2O_2 + 2OH^- \longrightarrow 2I^- + 2H_2O + O_2$

Choose the correct option.

a. H_2O_2 acts as oxidising agent in equations (A) and (B)
b. H_2O_2 acts as reducing agent in equations (A) and (B)
c. H_2O_2 act as oxidising and reducing agent respectively in equations (A) and (B)
d. H_2O_2 acts as reducing and oxidising agent respectively in equations (A) and (B).

6. Al_2O_3 was leached with alkali to get *X*. The solution of *X* on passing of gas *Y*, forms *Z*. *X*, *Y* and *Z* respectively are

a. $X = Na[Al(OH)_4]$, $Y = SO_2$, $Z = Al_2O_3$
b. $X - Al(OH)_3$, $Y - SO_2$, $Z - Al_2O_3 \cdot xH_2O$
c. $X = Al(OH)_3$, $Y = CO$, $Z = Al_2O_3$
d. $X = Na[Al(OH)_4]$, $Y = CO_2$, $Z = Al_2O_3 \cdot xH_2O$

7. The electrode potential of M^{2+}/M of 3*d*-series elements shows positive value for

a. Fe **b.** Co **c.** Zn **d.** Cu

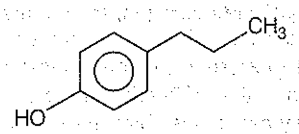
8. The major components in Gun metal are

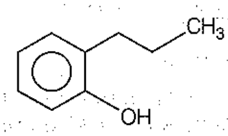
a. Cu, Sn and Zn **b.** Cu, Zn and Ni
c. Cu, Ni and Fe **d.** Al, Cu, Mg and Mn

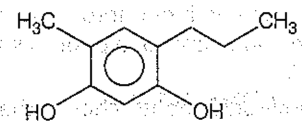
9. The gas released during anaerobic degradation of vegetation may lead to

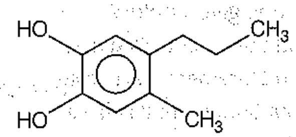
a. acid rain **b.** global warming and cancer
c. corrosion of metals **d.** ozone hole

10. Which of the following compound gives pink colour on reaction with phthalic anhydride in conc. H_2SO_4 followed by treatment with NaOH?

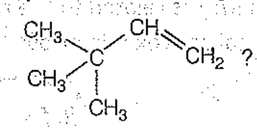
a.


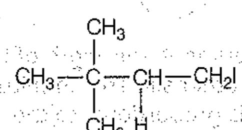
b.


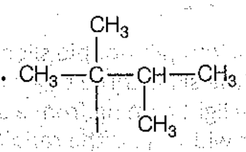
c.


d.


11. What is the major product formed by HI on reaction with



a.


b.


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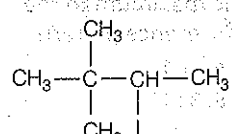
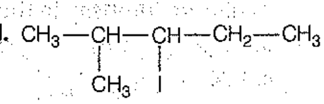


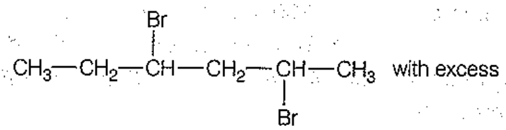
d.


12. Which of the following reagent is used for the following reaction ?

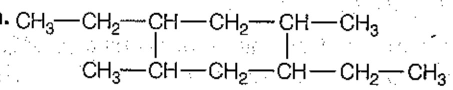
$CH_3CH_2CH_3 \xrightarrow{?} CH_3CH_2CHO$

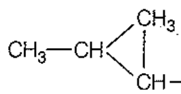
a. Copper at high temperature and pressure
b. Molybdenum oxide
c. Manganese acetate
d. Potassium permanganate

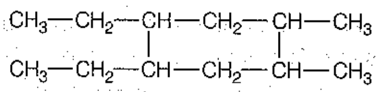
13. The product formed in the first step of the reaction of

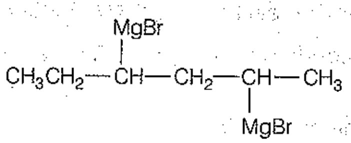


Mg/Et_2O ($Et = C_2H_5$) is

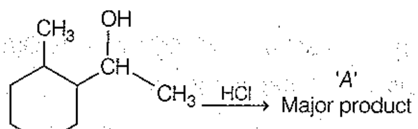
a. 

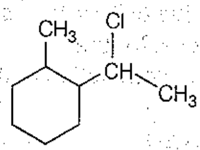
b. 

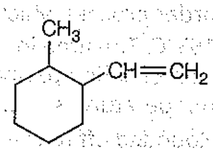
c. 

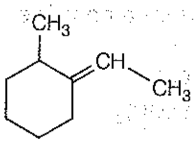
d. 

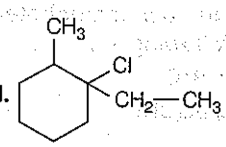
14. What is the final product (major) 'A' in the given reaction?



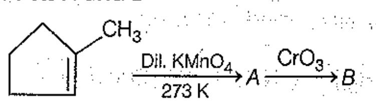
a. 

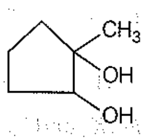
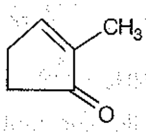
b. 

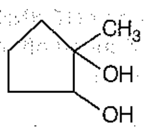
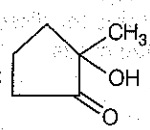
c. 

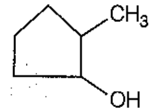
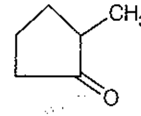
d. 

15. Identify products *A* and *B*



a. *A* :  *B* : 

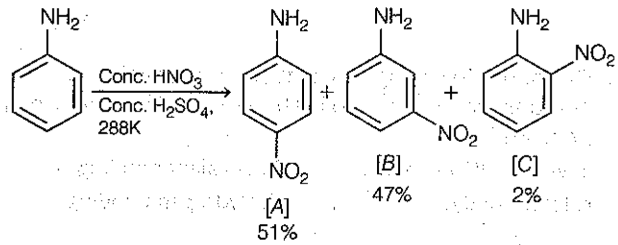
b. *A* :  *B* : 

c. *A* :  *B* : 

d. *A* : $OHC—CH_2CH_2CH_2—\overset{O}{\overset{||}{C}}—CH_3$

B : $HOOC—CH_2CH_2CH_2—\overset{O}{\overset{||}{C}}—CH_3$

16. In the following reaction the reason why *meta*-nitro product also formed is



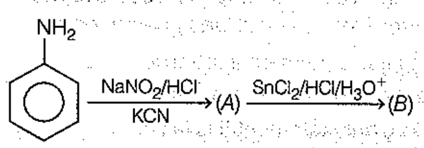
a. $—NH_2$ group is highly *meta*-directive

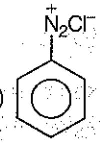
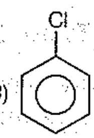
b. $—NO_2$ substitution always takes place at *meta*-position

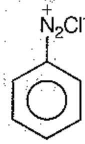
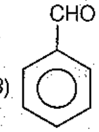
c. Formation of anilinium ion

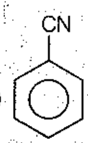
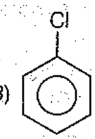
d. Low temperature

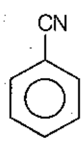
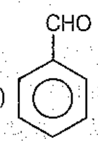
17. '*A*' and '*B*' in the following reaction are



a. (*A*)  (*B*) 

b. (*A*)  (*B*) 

c. (*A*)  (*B*) 

d. (*A*)  (*B*) 

18. Match List-I with List-II.

	List-I (Monomer Unit)		List-II (Polymer)
A.	Caprolactum	1.	Natural rubber
B.	2-chloro buta-1,3-diene	2.	Buna-N
C.	Isoprene	3.	Nylon-6
D.	Acrylonitrile	4.	Neoprene

Choose the correct answer from the options given below

	A	B	C	D		A	B	C	D
a.	1	2	3	4	**b.**	4	3	2	1
c.	2	1	4	3	**d.**	3	4	1	2

19. Out of the following, which type of interaction is responsible for the stabilisation of α-helix structure of proteins ?

a. van der Waals' forces **b.** Covalent bonding
c. Ionic bonding **d.** Hydrogen bonding

20. Given below are two statements.

Statement I Colourless cupric metaborate is reduced to cuprous metaborate in a luminous flame.

Statement II Cuprous metaborate is obtained by heating boric anhydride and copper sulphate in a non-luminous flame.

In the light of the above statements, choose the most appropriate answer from the options given below.

a. Both statement I and statement II are true.
b. Both statement I and statement II are false.
c. Statement I is true but statement II is false.
d. Statement I is false but statement II is true.

Section B : Numerical Type Questions

21. 4.5 g of compound A (MW = 90) was used to make 250 mL of its aqueous solution. The molarity of the solution in M is $x \times 10^{-1}$. The value of x is .... (Rounded off to the nearest integer).

22. The coordination number of an atom in a body centered cubic structure is ....

[Assume that the lattice is made up of atoms.]

23. A proton and a Li^{3+} nucleus are accelerated by the same potential. If λ_{Li} and λ_p denote the de-Broglie wavelengths of Li^{3+} and proton respectively, then the value of $\frac{\lambda Li}{\lambda_p}$ is $x \times 10^{-1}$ The value of x is ....

(Rounded off to the nearest integer)

(Mass of Li^{3+} = 8.3 mass of proton)

24. For the reaction, $A(g) \longrightarrow B(g)$, the value of the equilibrium constant at 300 K and 1 atm is equal to 100.0. The value of $\Delta_r G$ for the reaction at 300 K and 1 atm in J mol^{-1} is $-xR$, where x is .... (Rounded off to the nearest integer)

($R = 8.31$ J mol^{-1} K^{-1} and $\ln 10 = 2.3$)

25. When 9.45 g of $ClCH_2COOH$ is added to 500 mL of water, its freezing point drops by 0.5°C. The dissociation constant of $ClCH_2COOH$ is $x \times 10^{-3}$. The value of x is ....

(Rounded off to the nearest integer)

[$K_{f(H_2O)} = 1.86$ K kg mol^{-1}]

26. At 1990 K and 1 atm pressure, there are equal number of Cl_2 molecules and Cl atoms in the reaction mixture. The value of k_p for the reaction $Cl_2(g) \rightleftharpoons 2Cl(g)$ under the above conditions is $x \times 10^{-1}$ The value of x is ....

(Rounded off to the nearest integer)

27. The reaction of sulphur in alkaline medium is given below

$S_8(s) + aOH^-(aq) \longrightarrow bS^{2-}(aq) + cS_2O_3^{2-}(aq) + dH_2O(l)$

The value of 'a' is .... (Integer answer)

28. Gaseous cyclobutene isomerises to butadiene in a first order process which has a 'k' value of 3.3×10^{-4} s^{-1} at 153°C. The time in minutes it takes for the isomerisation to proceed 40 % to completion at this temperature is ....

(Rounded off to the nearest integer)

29. Number of amphoteric compounds among the following is ....

a. BeO **b.** BaO
c. $Be(OH)_2$ **d.** $Sr(OH)_2$

30. The stepwise formation of $[Cu(NH_3)_4]^{2+}$ is given below

$$Cu^{2+} + NH_3 \overset{K_1}{\rightleftharpoons} [Cu(NH_3)]^{2+}$$

$$[Cu(NH_3)]^{2+} + NH_3 \overset{K_2}{\rightleftharpoons} [Cu(NH_3)_2]^{2+}$$

$$[Cu(NH_3)_2]^{2+} + NH_3 \overset{K_3}{\rightleftharpoons} [Cu(NH_3)]^{2+}$$

$$[Cu(NH_3)_3]^{2+} + NH_3 \overset{K_4}{\rightleftharpoons} [Cu(NH_3)_4]^{2+}$$

The value of stability constants K_1, K_2, K_3 and K_4 are 10^4, 1.58×10^3, 5×10^2 and 10^2 respectively. The overall equilibrium constants for dissociation of $[Cu(NH_3)]^{2+}$] is $x \times 10^{-12}$. The value of x is .... (Rounded off to the nearest integer)

MATHEMATICS

Section A : Objective Type Questions

1. Let $f : R \to R$ be defined as $f(x) = 2x - 1$ and $g : R - \{1\} \to R$ be defined as $g(x) = \frac{x - (1/2)}{x - 1}$

Then, the composition function $f(g(x))$ is

a. one-one but not onto **b.** onto but not one-one

c. Neither one-one nor onto **d.** Both one-one and onto

2. Let p and q be two positive numbers, such that $p + q = 2$ and $p^4 + q^4 = 272$. Then, p and q are roots of the equation

a. $x^2 - 2x + 136 = 0$ **b.** $x^2 - 2x + 16 = 0$

c. $x^2 - 2x + 8 = 0$ **d.** $x^2 - 2x + 2 = 0$

3. The system of linear equations

$3x - 2y - kz = 10$

$2x - 4y - 2z = 6$

$x + 2y - z = 5m$

is inconsistent, if

a. $k \neq 3, m \neq \frac{4}{5}$ **b.** $k = 3, m = \frac{4}{5}$

c. $k = 3, m \neq \frac{4}{5}$ **d.** $k \neq 3, m \in R$

4. The value of $-{}^{15}C_1 + 2 \cdot {}^{15}C_2 - 3 \cdot {}^{15}C_3 + \ldots - 15 \cdot {}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \ldots + {}^{14}C_{11}$ is

a. $2^{16} - 1$ **b.** $2^{13} - 14$

c. $2^{13} - 13$ **d.** 2^{14}

5. If $e^{(\cos^2 x + \cos^4 x + \cos^6 x + \ldots \infty)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\frac{2 \sin x}{\sin x + \sqrt{3} \cos x}, \left(0 < x < \frac{\pi}{2}\right)$ is

a. $\frac{1}{2}$ **b.** $\sqrt{3}$ **c.** $\frac{3}{2}$ **d.** $2\sqrt{3}$

6. $\lim_{x \to 0} \frac{\int_0^{x^2} (\sin\sqrt{t})dt}{x^3}$ is equal to

a. 2/3 **b.** 3/2

c. 1/15 **d.** 0

7. The function

$f(x) = \frac{4x^3 - 3x^2}{6} - 2 \sin x + (2x - 1) \cos x$

a. increases in $\left[\frac{1}{2}, \infty\right)$ **b.** decreases in $\left[\frac{1}{2}, \infty\right)$

c. increases in $\left(-\infty, \frac{1}{2}\right]$ **d.** decreases in $\left(-\infty, \frac{1}{2}\right]$

8. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then, the number of ways, the committee can be formed, is

a. 1050 **b.** 1625

c. 560 **d.** 575

9. If $f : R \to R$ is a function defined by $f(x) = [x - 1]\cos\left(\frac{2x - 1}{2}\right)\pi$, where [·] denotes the greatest integer function, then f is

a. discontinuous only at $x = 1$

b. discontinuous at all integral values of x except at $x = 1$

c. continuous only at $x = 1$

d. continuous for every real x

10. If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a \sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$, where c is a constant of integration, then the ordered pair (a, b) is equal to

a. (3, 1) **b.** (1, 3)

c. (– 1, 3) **d.** (1, – 3)

11. The area (in sq. units) of the part of the circle $x^2 + y^2 = 36$, which is outside the parabola $y^2 = 9x$, is

a. $24\pi + 3\sqrt{3}$ **b.** $24\pi - 3\sqrt{3}$

c. $12\pi + 3\sqrt{3}$ **d.** $12\pi - 3\sqrt{3}$

12. The population $P = P(t)$ at time t of a certain species follows the differential equation $\frac{dP}{dt} = 0.5P - 450$. If $P(0) = 850$, then the time at which population becomes zero is

a. $\log_e 9$ **b.** $\frac{1}{2}\log_e 18$

c. $\log_e 18$ **d.** $2\log_e 18$

13. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1,1), (2, 2) and (4, 4), respectively. Then, which of these stones is / are on the path of the man ?

a. A only **b.** B only

c. C only **d.** All the three

14. The locus of the mid-point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola, is another parabola whose directrix is

a. $x = a$ **b.** $x = -\frac{a}{2}$

c. $x = 0$ **d.** $x = \frac{a}{2}$

15. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is

a. 0 **b.** $2t^3$ **c.** $-t^3$ **d.** $-2t^3$

16. The equation of the plane passing through the point (1, 2, –3) and perpendicular to the planes $3x + y - 2z = 5$ and $2x - 5y - z = 7$, is

a. $6x - 5y + 2z + 10 = 0$

b. $11x + y + 17z + 38 = 0$

c. $6x - 5y - 2z - 2 = 0$

d. $3x - 10y + 2z + 11 = 0$

17. The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane $x + y + z = 17$ is

a. $2\sqrt{19}$ **b.** $19\sqrt{2}$

c. $\sqrt{38}$ **d.** 38

18. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is

a. $\frac{1}{32}$ **b.** $\frac{3}{16}$

c. $\frac{5}{16}$ **d.** $\frac{1}{2}$

19. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in metres) is

a. 25 **b.** 30

c. $20\sqrt{3}$ **d.** $25\sqrt{3}$

20. The statement among the following that is a tautology is

a. $A \wedge (A \vee B)$ **b.** $A \vee (A \wedge B)$

c. $[A \wedge (A \to B)] \to B$ **d.** $B \to [A \wedge (A \to B)]$

Section B : Numerical Type Questions

21. If the least and the largest real values of α, for which the equation $z + \alpha |z - 1| + 2i = 0$ ($z \in C$ and $i = \sqrt{-1}$) has a solution, are p and q respectively, then $4(p^2 + q^2)$ is equal to ....

22. Let $B_i (i = 1, 2, 3)$ be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let P be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)P = \alpha\beta$ and $(\beta - 3\gamma)P = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0,1)). Then, $\frac{P(B_1)}{P(B_3)}$ is equal to ....

23. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in R$. If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to ....

24. Let M be any 3×3 matrix with entries from the set {0, 1, 2}. The maximum number of such matrices, for which the sum of diagonal elements of $M^T M$ is seven, is ....

25. Let $A = \{n \in N : n$ is a 3-digit number$\}$

$B = \{9k + 2 : k \in N\}$

and $C = \{9k + l : k \in N\}$ for some $l (0 < l < 9)$

If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then l is equal to ....

26. The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is ....

27. If $\int_{-a}^{a} (|x| + |x - 2|) dx = 22, (a > 2)$ and $[x]$ denotes the greatest integer $\leq x$, then $\int_{a}^{-a} (x + [x]) \, dx$ is equal to ....

28. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C', whose centre is at (2, 1), then its radius is ....

29. Let three vectors **a**, **b** and **c** be such that **c** is coplanar with **a** and **b**, **a**·**c** = 7 and **b** is perpendicular to **c**, where $\mathbf{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\mathbf{b} = 2\hat{i} + \hat{k}$, then the value of $2|\mathbf{a} + \mathbf{b} + \mathbf{c}|^2$ is ....

30. $\lim_{n \to \infty} \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1 + r + r^2}\right)\right\}$ is equal to ....

Answers

Physics

1. (c)	2. (b)	3. (*)	4. (d)	5. (a)	6. (d)	7. (c)	8. (a)	9. (d)	10. (d)
11. (b)	12. (c)	13. (c)	14. (d)	15. (c)	16. (a)	17. (d)	18. (c)	19. (d)	20. (a)
21. (25)	22. (75)	23. (1)	24. (25600)	25. (25)	26. (2000)	27. (25)	28. (25)	29. (15)	30. (440)

Chemistry

1. (a)	2. (d)	3. (a)	4. (d)	5. (b)	6. (d)	7. (d)	8. (a)	9. (b)	10. (b)
11. (b)	12. (b)	13. (d)	14. (d)	15. (d)	16. (c)	17. (d)	18. (d)	19. (d)	20. (b)
21. (2)	22. (8)	23. (2)	24. (1380)	25. (34.4)	26. (5)	27. (12)	28. (26)	29. (2)	30. (1.26)

Mathematics

1. (a)	2. (b)	3. (c)	4. (b)	5. (a)	6. (a)	7. (a)	8. (b)	9. (d)	10. (b)
11. (b)	12. (d)	13. (b)	14. (c)	15. (d)	16. (b)	17. (c)	18. (d)	19. (d)	20. (c)
21. (10)	22. (6)	23. (17)	24. (540)	25. (5)	26. (9)	27. (3)	28. (3)	29. (75)	30. (1)

Note *(*) None of the option is correct.*

Solutions

PHYSICS

1. *(c)* Given, work done, $W = \alpha \cdot \beta^2 e^{-\frac{x^2}{\alpha k T}}$

where, k is Boltzmann constant,

T is temperature and x is displacement.

We know that, $\frac{x^2}{\alpha k T}$ is a dimensionless quantity.

$$\therefore \quad \left[\frac{x^2}{\alpha k T}\right] = [M^0L^0T^0] \Rightarrow [\alpha] = \frac{[x^2]}{[k][T]}$$

$$\Rightarrow \quad [\alpha] = \frac{[L^2]}{[k][T]} \quad \ldots \text{(i)}$$

Since, dimensions of k are

$$[k] = [M^1L^2T^{-2}K^{-1}] \quad \ldots \text{(ii)}$$

Dimensions of temperature are

$$[T] = [K] \quad \ldots \text{(iii)}$$

Substituting Eqs. (ii) and (iii) in Eq. (i), we get

$$[\alpha] = \frac{[L^2]}{[M^1L^2T^{-2}K^{-1}][K]}$$

$$[\alpha] = [M^{-1}T^2]$$

According to dimensional analysis,

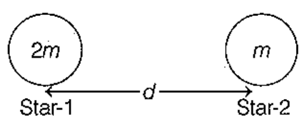
$$[W] = [\alpha\beta^2]$$

$$\Rightarrow \quad [\beta^2] = \frac{[W]}{[\alpha]}$$

$$\Rightarrow \quad [\beta^2] = \frac{[M^1L^2T^{-2}]}{[M^{-1}T^2]} = [M^2L^2T^{-4}]$$

$$\Rightarrow \quad [\beta] = [MLT^{-2}]$$

2. *(b)* The given situation is shown below



The gravitational force between these two stars provide the required centripetal force for rotation in a circle about their common centre.

Assuming $2m$ at origin, the centre of mass of the system lies at

$$x = \frac{2m \times 0 + m \times d}{2m + m} = \frac{d}{3}$$

Hence, $\quad F_G = F_C$

where, F_G is gravitational force between them and F_C is centripetal force.

$$\Rightarrow \quad \frac{Gm_1m_2}{r^2} = 2m\omega^2 x$$

$$\Rightarrow \quad \frac{G(2m)(m)}{d^2} = 2m\omega^2 \times \frac{d}{3} \Rightarrow \omega^2 = \frac{3Gm}{d^3}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{3Gm}{d^3}} \quad \ldots \text{(i)}$$

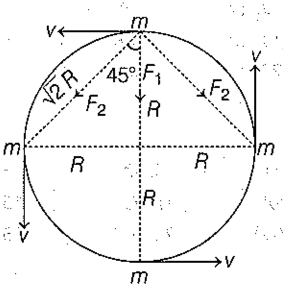
We know that, $\quad \omega = \frac{2\pi}{T}$

$$\therefore \quad T = \frac{2\pi}{\omega}$$

$$\Rightarrow \quad T = \frac{2\pi}{\sqrt{\frac{3Gm}{d^3}}}$$

$$= 2\pi\sqrt{\frac{d^3}{3Gm}} \quad \text{[using Eq. (i)]}$$

3. *(*)*



Given, $m = 1\,\text{kg}$, $R = 1\,\text{m}$

We know that,

$$F = \frac{Gm_1m_2}{r^2}$$

$$\because \quad F_1 = \frac{Gmm}{(2R)^2} = \frac{Gm^2}{4R^2}$$

and $\quad F_2 = \frac{Gmm}{(\sqrt{2}R)^2} = \frac{Gm^2}{2R^2}$

Net force on one particle,

$$F_{\text{net}} = F_1 + F_2\cos 45° + F_2 \cos 45°$$

$$= F_1 + 2F_2\cos 45°$$

$$= \frac{Gm^2}{4R^2} + 2\left(\frac{Gm^2}{2R^2}\right)\cdot\frac{1}{\sqrt{2}}$$

$$= \frac{Gm^2}{4R^2} + \frac{Gm^2}{\sqrt{2}R^2}$$

$$= \frac{Gm^2}{R^2}\left[\frac{1}{4} + \frac{1}{\sqrt{2}}\right]$$

As the gravitational force provides the necessary centripetal force, so

$$F_{\text{net}} = F_C = \frac{mv^2}{R}$$

Here, F_C = centripetal force.

$$\Rightarrow \quad \frac{Gm^2}{R^2}\left[\frac{1}{4} + \frac{1}{\sqrt{2}}\right] = \frac{mv^2}{R}$$

$$\Rightarrow \quad v = \frac{1}{2}\sqrt{\frac{Gm}{R}(1 + 2\sqrt{2})}$$

$$\Rightarrow \quad v = \frac{1}{2}\sqrt{G(1 + 2\sqrt{2})}$$

4. *(d)* Let M and R be the mass and radius of four bodies. Then, as per question, their moment of inertia are

$$I_1 = \frac{MR^2}{2}, I_2 = \frac{MR^2}{2}, I_3 = \frac{MR^2}{2},$$

$$I_4 = \frac{2}{5}MR^2$$

Clearly, $I_1 = I_2 = I_3 > I_4$

5. *(a)* Given, period of revolution of first satellite,

$$T_1 = 1\text{h}$$

Period of revolution of second satellite,

$$T_2 = 8\text{ h}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{1}{8}$$

We know that, $\omega = \frac{2\pi}{T}$

$$\Rightarrow \quad \omega \propto \frac{1}{T}$$

$$\because \quad \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1} \Rightarrow \frac{\omega_1}{\omega_2} = \frac{8}{1}$$

or $\quad \omega_1 : \omega_2 = 8:1$

6. *(d)* We know that, $\gamma = 3\alpha$ ... (i)

where, α is the coefficient of linear expansion and γ is the coefficient of volume expansion.

We know that,

$$\frac{\Delta V}{V} = \gamma \Delta T$$

$$\Rightarrow \quad \frac{\Delta V}{V} = 3\alpha \Delta T \quad \text{[from Eq. (i)]}$$

$$\Delta V = 3a^3 \alpha \Delta T \quad [\because \text{volume of cube} = a^3]$$

7. *(c)* We know that,

$$Y = 3K(1-2\sigma)$$

$$\Rightarrow \quad \sigma = \frac{1}{2}\left(1 - \frac{Y}{3K}\right) \quad \text{...(i)}$$

Also, $\quad Y = 2\eta(1+\sigma)$

$$\Rightarrow \quad \sigma = \frac{Y}{2\eta} - 1 \quad \text{... (ii)}$$

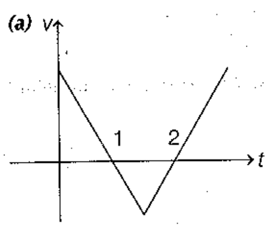
On comparing Eqs. (i) and (ii), we get

$$\left(1 - \frac{Y}{3K}\right)\frac{1}{2} = \frac{Y}{2\eta} - 1$$

On solving, we get

$$K = \frac{\eta Y}{9\eta - 3Y}\text{ N/m}^2$$

8. *(a)*



From the graph for first line, the slope is negative and intercept is positive.

So, equation of line is

$$v = -mt + c$$

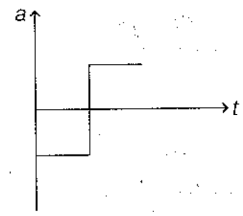
$$\Rightarrow \quad a_1 = \frac{dv}{dt} = -m$$

Similarly, for second line, the slope is positive and intercept is negative, so equation of line is

$$v = mt - c$$

$$\Rightarrow \quad a_2 = \frac{dv}{dt} = m$$

$\therefore$ The corresponding acceleration-time graph as shown below



Hence, option (a) is correct.

9. *(d)* As, AB is isothermal process, so work done by isothermal process is given by

$$W_{AB} = nRT \ln\left(\frac{V_2}{V_1}\right)$$

$$W_{AB} = nRT \ln\left(\frac{2V_1}{V_1}\right) = nRT \ln 2$$

As, BC is isobaric process, so work done by isobaric process is given by

$$W_{BC} = p\Delta V = p_2(V_1 - V_2)$$

$$= p_2\left(\frac{V_2}{2} - V_2\right) = -\frac{p_2V_2}{2}$$

$$= \frac{-nRT}{2} \quad \left[\begin{array}{l}\because pV = nRT \\ \therefore p_2V_2 = nRT\end{array}\right]$$

As, CA is isochoric process, so work done by isochoric process is given by, $W_{CA} = 0$ $\quad [\because \Delta V = 0]$

Total work done in the cycle $ABCA$,

$$W_{ABCA} = W_{AB} + W_{BC} + W_{CA}$$

$$= nRT \ln 2 + \left(\frac{-nRT}{2}\right) + 0$$

$$= nRT\left(\ln 2 - \frac{1}{2}\right)$$

10. *(d)* We know that, in isothermal process, $\Delta T = 0$

In isochoric process, $\Delta V = 0$

In adiabatic process, $\Delta Q = 0$

In isobaric process, $\Delta p = 0$

So, the correct match is,

A $\rightarrow$ 2, B $\rightarrow$ 3, C $\rightarrow$ 4, D $\rightarrow$ 1.

11. *(b)* Given, initial amplitude = A

Velocity at mean position, $v = A\omega$

Applying conservation of momentum at mean position, we get

$$M_1v_1 = M_2v_2$$

$$MA\omega = (M+m)v'$$

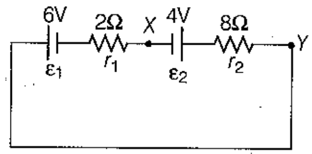
$$\Rightarrow \quad v' = \frac{MA\omega}{M+m} = \frac{MA\sqrt{\frac{k}{M}}}{M+m}$$

$$\therefore \quad v' = A'\omega' = A'\sqrt{\frac{k}{M+m}}$$

$$\Rightarrow \quad A' = \frac{MA\sqrt{\frac{k}{M}}}{M+m} \times \sqrt{\frac{M+m}{k}}$$

$$A' = \sqrt{\frac{M}{M+m}}\,A$$

12. *(c)* The circuit can be shown as below



The current through the circuit,

$$I = \frac{\varepsilon_1 - \varepsilon_2}{r_1 + r_2} = \frac{6-4}{10} = \frac{1}{5}\text{ A}$$

∴ Potential difference across points X and Y is

$$V_{XY} = E_2 + Ir_2 = 4 + \frac{1}{5} \times 8 = 5.6\text{ V}$$

13. *(c)* Given, $i = \alpha_0 t + \beta t^2$

where, $\alpha_0 = 20\text{ A/s}, \beta = 8\text{ A/s}^2$

We know that, $i = \frac{dq}{dt}$

$$\Rightarrow \quad \frac{dq}{dt} = i = \alpha_0 t + \beta t^2 = 20t + 8t^2$$

$$\Rightarrow \quad dq = (20t + 8t^2)dt$$

On integrating both sides, we get

$$\int_0^q dq = \int_0^{15} (20t + 8t^2)dt$$

$$q = \left[\frac{20t^2}{2} + \frac{8t^3}{3}\right]_0^{15} = 10 \times (15)^2 + \frac{8}{3} \times (15)^3$$

$$\therefore \quad q = 11250\text{C}$$

14. *(d)* Given, $C_1 = C_2 = C$

When both capacitors are connected in series, their equivalent capacitance will be

$$\frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$$

$$\Rightarrow \quad C_s = \frac{C}{2}$$

When both capacitors are connected in parallel, their equivalent capacitance will be

$$C_p = C + C = 2C$$

∴ The ratio of equivalent capacitance in series and parallel combination is

$$\frac{C_s}{C_p} = \frac{C/2}{2C} = \frac{1}{4}$$

$$\Rightarrow \quad C_s : C_p = 1 : 4$$

15. *(c)* We can replace − Q charge at origin by + Q and − 2Q. Now, due to + Q charge at every corner of cube, electric field at centre of cube is zero. So, net electric field at centre is only due to − 2Q charge at origin. Vector form of electric field strength,

$$\mathbf{E} = \frac{Kq\mathbf{r}}{r^3}$$

Here, position vector, $r = \frac{a}{2}(\hat{x} + \hat{y} + \hat{z})$

$$\Rightarrow \quad |\mathbf{r}| = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{\sqrt{3}a}{2}$$

$$\Rightarrow \quad \mathbf{E} = \frac{1}{4\pi\varepsilon_0} \times \frac{(-2Q)\,a/2}{\left(\frac{a}{2}\sqrt{3}\right)^3} (\hat{x} + \hat{y} + \hat{z})$$

$$\mathbf{E} = \frac{-2Q(\hat{x} + \hat{y} + \hat{z})}{3\sqrt{3}\,\pi\, a^2 \varepsilon_0}$$

16. *(a)* Given, emitter current, $I_E = 4$ mA

Collector current, $I_C = 3.5$ mA

Current gain in common base amplifier,

$$\alpha = \frac{I_C}{I_E}$$

$$\Rightarrow \quad \alpha = \frac{3.5}{4} = \frac{7}{8}$$

Also, current gain in common emitter amplifier,

$$\beta = \frac{\alpha}{1-\alpha}$$

$$\Rightarrow \quad \beta = \frac{7/8}{1 - 7/8}$$

$$\beta = 7$$

17. *(d)* In transition A, hydrogen atom comes from higher energy state $n = \infty$ to lower energy state $n = 1$. Hence, transition A represents series limit of Lyman series. In transition B, hydrogen atom comes from higher energy state $n = 5$ to lower energy state $n = 2$. Hence, transition B represents 3rd line of Balmer series.

In transition C, hydrogen atom comes from higher energy state $n = 5$ to lower energy state $n = 3$. Hence, transition C represents 2nd line of Paschen series.

Hence, option (d) is correct.

18. *(c)* As we know, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$

If linear momenta of two photons are equal, then their wavelengths is also equal.

Also, if the wavelength is decreased, then the momentum and energy of photon will increase.

Hence, option(c) is correct.

19. *(d)* For convex mirror, the focal length (f) and radius of curvature (r) are related as $f = +\frac{r}{2}$.

20. *(a)* Given, amplitude ∝ width of slit

$$\Rightarrow \quad A_2 = 3A_1$$

We know that,

$$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

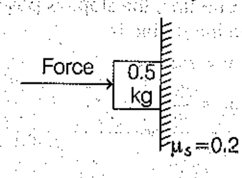
∵ Intensity, $I \propto A^2$

$$\therefore \quad \frac{I_{\max}}{I_{\min}} = \frac{(A_1 + A_2)^2}{(|A_1 - A_2|)^2}$$

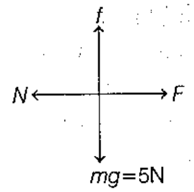
$$= \left(\frac{A_1 + 3A_1}{|A_1 - 3A_1|}\right)^2 = \left(\frac{4A_1}{2A_1}\right)^2 = \frac{4}{1}$$

$$\therefore \quad I_{\max} : I_{\min} = 4 : 1$$

21. *(25)* Given, coefficient of static friction, $\mu_s = 0.2$



Various forces acting on block are shown below



Frictional force $\le mg$

$\Rightarrow \quad N \times 0.2 \le 5$

$\Rightarrow \quad N \le 25$

$\therefore$ Magnitude of horizontal force, $F = N = 25$ N

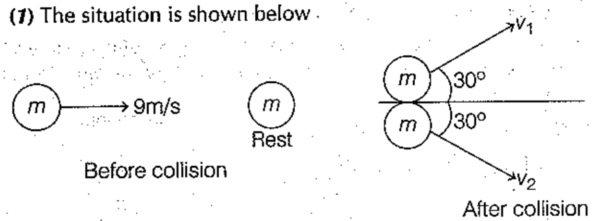
22. *(75)* Given, $I_0 = 100$ lumens

When analyser is rotated through an angle θ, the intensity of light will becomes

$$I = I_0 \cos^2\theta = 100 \times \cos^2 30^\circ$$

$$= 100 \times \left(\frac{\sqrt{3}}{2}\right)^2 = 75 \text{ lumens}$$

23. *(1)* The situation is shown below



Using conservation of linear momentum in y-direction,

$$p_i = p_f$$

As, $p_i = 0$

and $p_f = mv_1 \sin 30^\circ - mv_2 \sin 30^\circ$

$\Rightarrow \quad 0 = m \times \frac{1}{2} v_1 - m \times \frac{1}{2} v_2$

$\Rightarrow \quad v_1 = v_2$ or $v_1 : v_2 = 1 : 1$

Since, $v_1 : v_2 = x : y$ (given)

$\therefore \quad x = 1$

24. *(25600)* According to Pascal's law,

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Initially, $\quad \frac{100g}{A_1} = \frac{mg}{A_2} \quad$ ... (i)

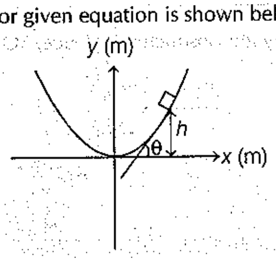
Finally, $\quad \frac{Mg}{16A_1} = \frac{mg}{\left(\frac{A_2}{16}\right)} \quad$ ... (ii)

On dividing Eqs. (i) by (ii), we get

$$\frac{100 \times 16}{M} = \frac{1}{16}$$

$\therefore \quad M = 25600$ kg

25. *(25)* The graph for given equation is shown below



At maximum height, the slope of tangent drawn,

$$\tan\theta = \frac{dy}{dx} = \frac{2x}{4} = \frac{x}{2} \qquad \left[\because y = \frac{x^2}{4}\right]$$

$\Rightarrow \quad 0.5 = \frac{x}{2} \qquad (\because \mu = \tan\theta)$

$\Rightarrow \quad x = 1$ m

$\therefore \quad y = \frac{x^2}{4} = \frac{1}{4} = 0.25 \text{ m} = 25 \text{ cm}$

26. *(2000)* Given, $L = 2 \times 10^{-4}$ H, $R = 6.28\,\Omega$,

$f_0 = 10 \text{ MHz} = 10 \times 10^6$ Hz

$\therefore$ Quality factor $= \omega_0 \frac{L}{R} = 2\pi f_0 \frac{L}{R}$

$$= 2\pi \times 10 \times 10^6 \times \frac{2 \times 10^{-4}}{6.28}$$

$$= 2 \times 10^3 = 2000$$

27. *(25)* Given, audio signal,

$V_m = 20 \sin 2\pi(1500t) \quad$ ... (i)

Carrier signal, $V_c = 80 \sin 2\pi(100000t) \quad$ ... (ii)

We know that, modulation index,

$$m_f = \frac{A_m}{A_c}$$

From Eqs. (i) and (ii), we get

$$A_m = 20, A_c = 80$$

Percentage of modulation index,

$$m_f = \frac{A_m}{A_c} \times 100 = \frac{20}{80} \times 100 = 25\%$$

28. *(25)* Given, resistance, $R = 2\text{ k}\Omega = 2 \times 10^3\,\Omega$

In Zener breakdown,

$$i = \frac{V}{R} = \frac{5}{2 \times 10^3} = 2.5 \times 10^{-3}$$

$\because \quad x \times 10^{-4} = 25 \times 10^{-4}$

$\therefore \quad x = 25$

29. *(15)* Given, $\mu_r = \varepsilon_r = 2$

where, μ_r is relative permeability,

ε_r is relative permittivity.

Speed of electromagnetic wave v is given by

$$v = \frac{c}{n}$$

where, n = refractive index $= \sqrt{\mu_r \varepsilon_r} = \sqrt{4} = 2$

$\Rightarrow \quad v = \frac{3 \times 10^8}{2} = 15 \times 10^7$ m/s

$\because \quad x \times 10^7 = 15 \times 10^7$

$\Rightarrow \quad x = 15$

30. *(440)* In a transformer,

$$\frac{N_p}{N_s} = \frac{V_p}{V_s}$$

where, N_p = number of turns in primary circuit,

N_s = number of turns in secondary circuit = 24,

V_p = potential of primary circuit = 220V

and V_s = potential of secondary circuit = 12 V

$\Rightarrow \quad \frac{N_p}{24} = \frac{220}{12}$

$\Rightarrow \quad N_p = 440$

CHEMISTRY

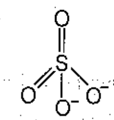
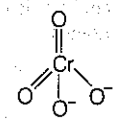
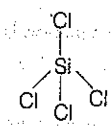
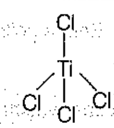
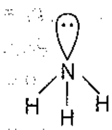
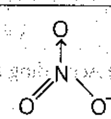
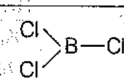
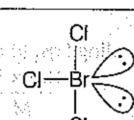
1. ***(a)*** Isostructural compounds are those compounds which have same structure as well as same hybridisation.

Formula for find hybridisation : H = (lp + σ + coordinate bond)

lp = Lone pair

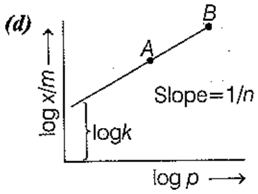
bp = Bond pair

σ = Sigma bond

S. No.	Molecule	H (Steric number)	Hybridisation	B.P.	L.P.	Shape
(A)	SO_4^{2-}	4	sp^3	4	0	 Tetrahedral
	CrO_4^{2-}	4	sp^3	4	0	 Tetrahedral
(B)	$SiCl_4$	4	sp^3	4	0	 Tetrahedral
	$TiCl_4$	4	sp^3	4	0	 Tetrahedral
(C)	NH_3	4	sp^3	3	1	 Trigonal pyramidal
	NO_3^-	4	sp^2	3	0	 Trigonal planar
(D)	BCl_3	3	sp^2	3	0	 Trigonal planar
	$BrCl_3$	5	sp^3d	3	2	 T - shape

If number of sigma bond (σ), co-ordinate bond and lone pair are same for given pairs, they are isostructural. Hence, SO_4^{2-}, CrO_4^{2-}, $SiCl_4$ and $TiCl_3$ have tetrahedral shape and are isostructural.

2. *(d)*



Given graph is Freundlich adsorption isotherm.

According to equation, $\frac{x}{m} = kp^{\frac{1}{n}}$ ...(i)

where, $\frac{x}{m}$ = amount of gas

k and n = constants depending upon nature of adsorbate and absorbent.

Now, taking log both side in Eq. (i), we get

$\log\frac{x}{m} = \log k + \frac{1}{n}\log p$

For straight line, comparing above equation to $y = mx + c$

So, $m = \text{slope} = \frac{1}{n}$

hence, $0 \le \frac{1}{n} \le 1$

It value lies in between 0 to 1.

Hence, $\frac{1}{n}$ with $\left(\frac{1}{n} = 0 \text{ to } 1\right)$.

3. *(a)* On moving left to right in a period of the periodic table, Ionisation energy (I.E.) increases due to increase in effective nuclear charge i.e. Z_{eff}.

But due to extra stability of fully filled and half-filled electronic configuration of Mg and P required ionisation enthalpy is more from neighbouring elements.

i.e. First ionisation enthalpy order is Al < Mg < Si < S < P.

4. *(d)* The chemical formulas of given ore in options are as follows

Sphalerite ore : ZnS

Calamine ore : $ZnCO_3$

Siderite ore : $FeCO_3$

Malachite ore : $Cu(OH)_2 \cdot CuCO_3$

Sphalerite ore containing ZnS is concentrated using group is cyanide salt (NaCN) as a depressant by froth flotation method.

5. *(b)* In equation (A), HOCl undergoes reduction in presence of H_2O_2.

Reduction

$$\overset{(+1)}{HOCl} + H_2O_2 \longrightarrow H_3O^+ + \overset{(-1)}{Cl^-} + O_2$$

Here, oxidation state of Cl changes from +1 to −1 (i.e. reduces)

$\therefore$ H_2O_2 act as reducing agent I_2 reduces to I^- in presence of H_2O_2.

In equation (B),

Reduction

$$\overset{0}{I_2} + \overset{+1}{H_2O_2} + 2OH^- \longrightarrow \overset{-1}{2I^-} + \overset{+2}{2H_2O} + O_2$$

Here oxidation state of iodine decreases (from 0 to −1)

$\therefore$ H_2O_2 act as reducing agent in both the equations.

6. *(d)* Al_2O_3 (aluminium oxide) was leached with alkali to get $Na[Al(OH)_4](X)$ (sodium aluminate). The solution of sodium aluminate when passed through gas Y, i.e. carbon dioxide (CO_2) forms ($Al_2O_3 \cdot xH_2O$) (Z).

Complete reactions are as follows

$$\underset{\text{Aluminium oxide}}{Al_2O_3} + \underset{\text{Sodium hydroxide}}{2NaOH(aq)} + 3H_2O(l) \longrightarrow$$

$$\underset{(X)}{2Na[Al(OH)_4](aq)} \xrightarrow{+CO_2(Y)} \underset{(Z)}{Al_2O_3 \cdot xH_2O} + \underset{\text{Sodium bicarbonate}}{2NaHCO_3}$$

Hence, $X = Na[Al(OH)_4]$

$Y = CO_2$

$Z = Al_2O_3 \cdot xH_2O$

7. *(d)* In the electrode potential series, only copper have positive value for electrode potential because copper has lower tendency than hydrogen to form ions. So, if standard hydrogen electrode ($E_{Cell} = 0$) is connected to copper half-cell, the copper with be relatively less negative or less number of electrons.

$E^\circ_{(Cu^{2+}/Cu)} = +0.34$ V; $E^\circ_{(Fe^{2+}/Fe)} = -0.41$ V

$E^\circ_{Co^{+2}/Co} = -0.28$ V; $E^\circ_{(Zn^{2+}/Zn)} = -0.76$ V

$\therefore$ Electrode potential of Cu $E^\circ_{Cu^{+2}/Cu}$ show positive value.

8. *(a)* 'Gun metal' also called as red brass (in USA). It is greyish in colour and originally used for making guns. The major components of 'Gun metal' are

Cu = 87%

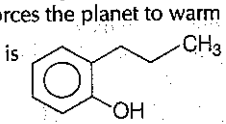
Zn = 3%

Sn = 10%

It is also used in manufacturing of gears and bearing that are to be subjected to heavy loads and low speeds.

9. *(b)* Methane (CH_4) gas is evolved due to anaerobic degradation of vegetation which causes global warming and cancer. It is a heat trapping gas that forces the planet to warm drastically and quickly.

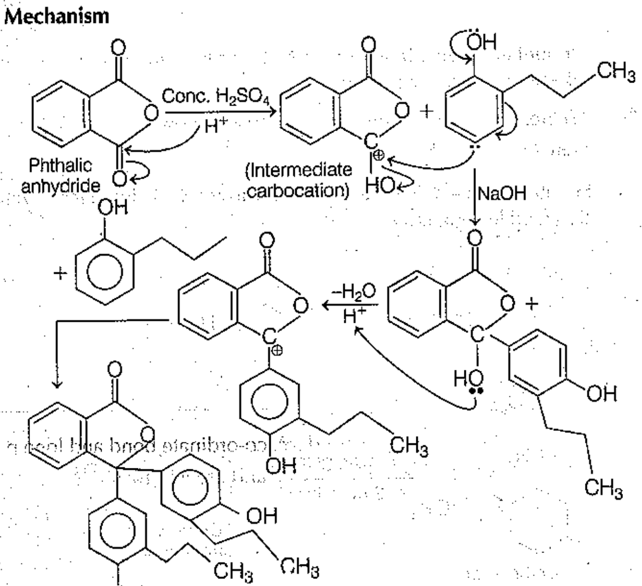
10. *(b)* Correct answer is



2-propylphenol

Firstly phthalic anhydride in presence of conc.H_2SO_4 undergoes protonation to give an intermediate carbocation. This carbocation reacts with 2-propylphenol in presence of NaOH to give pink colour compound.

Mechanism



(Pink colour)

2-propylphenol gives pink colour on reaction with phthalic anhydride in conc. H_2SO_4 followed by treatment with NaOH.

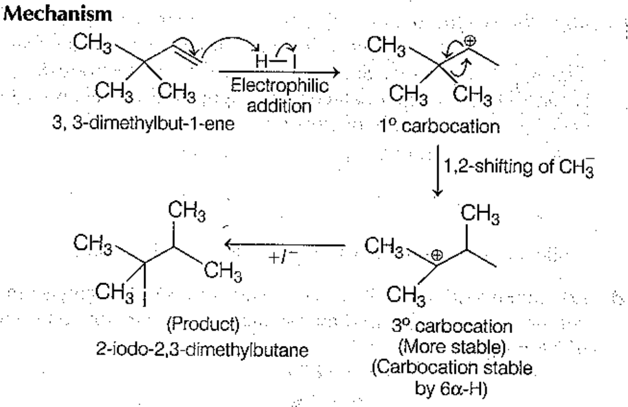
11. *(b)* The major product formed is 2-iodo-2, 3-dimethylbutane. Steps involved in the reaction are as follows :

Step 1 It is electrophilic addition reaction, π-bond of alkene attack H^+ ion of HI and form more stable carbocation.

Step 2 Formation of more stable 3° carbocation take place by 1, 2 shifting of —Me group.

Step 3 Direct addition of $I^{\ominus}$ ion .

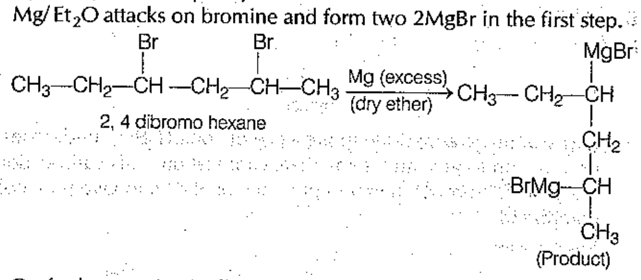
Mechanism



12. *(b)* Molybdenum oxide (Mo_2O_3) is used for oxidising alkanes to aldehyde. It used to manufacture molybdenum metal, which serves as an additive to steel and corrosive resistant alloys.

$$\underset{\text{Propane}}{CH_3—CH_2—CH_3} \xrightarrow[\text{Oxidising agent}]{Mo_2O_3} \underset{\text{Propanal}}{CH_3—CH_2—CHO}$$

13. *(d)* Here, in first step only one mole of Mg/Et_2O attacks on bromine and form two 2MgBr in the first step.

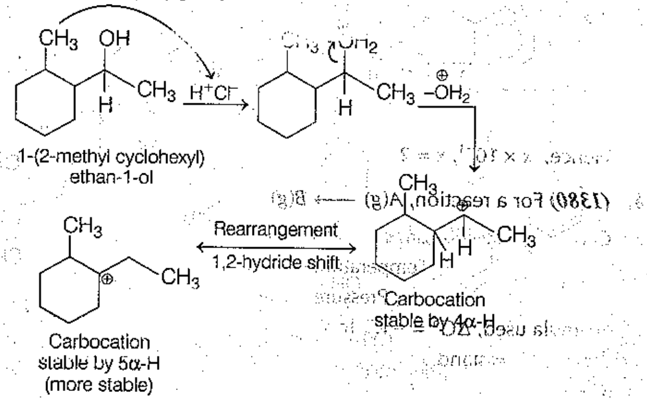


On further moving in the reaction, two MgBr are eliminated to form alkene in respective positions.

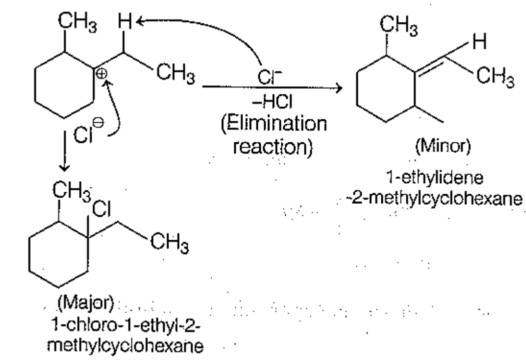
14. *(d)* Steps involved in this reaction are as follows

Step 1 $HCl \longrightarrow H^+ + Cl^-$

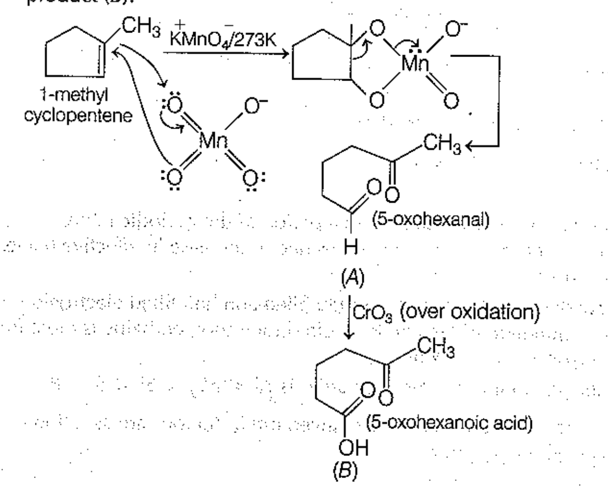
H^+ attacks on —OH (lone pair) and formed $\overset{\oplus}{O}H_2$ ion. Here, $\overset{\oplus}{O}H_2$ is the good leaving group.



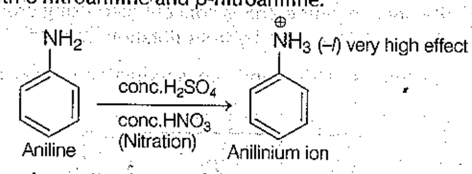
Step 2 Elimination reaction always give minor product than substitution reaction.



15. *(d)* In first step, $KMnO_4$ (dil.) help to break alkene in ketone and aldehyde and in second step, CrO_3 is used for selective over oxidation of aldehyde only. CrO_3 will form aldehyde to acid in product *(B)*.



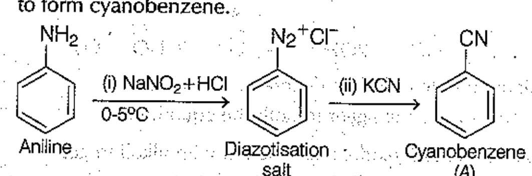
16. *(c)* Aniline on protonation forms anilinium ion, which is *meta*-directing. So, a considerable amount of *meta* product is formed alongwith *o*-nitroaniline and *p*-nitroaniline.



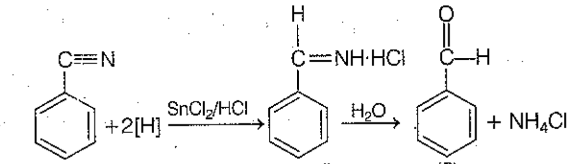
Nitrating mixture is mixture of conc. HNO_3 and a conc. H_2SO_4. When aniline is reacted with nitrating mixture *ortho*, *meta* and *para* nitroanilines are obtained.

Aniline being basic in nature, reacts with acids to form anilinium ion which is *meta* directing.

17. *(d)* **Step 1** In step 1, $NaNO_2$ + HCl, 0-5°C used for diazotisation reaction. It will form diazonium salt. Further, it will react with KCN to form cyanobenzene.

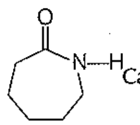


Step 2 In step 2, $SnCl_2$ and HCl is a Stephen's reduction reagent. Cyanobenzene reduced to benzaldehyde by $SnCl_2/HCl$.



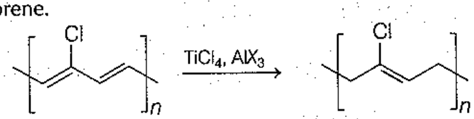
Cyanobenzene Intermediate (B) Benzaldehyde

18. *(d)*

(a) 

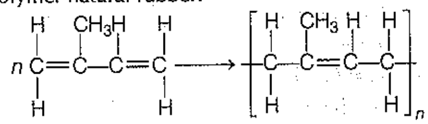
Caprolactum is the monomeric unit of polymer nylon-6 $\left[HN-(CH_2)_5-\overset{O}{\overset{\|}{C}} \right]_n$

(b) 2-chlorobuta -1, 3-diene is the monomeric unit of polymer neoprene.



2-chlorobuta-1,3-diene Neoprene

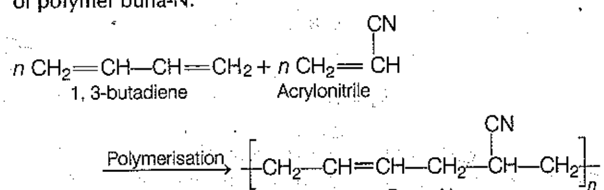
(c) 2-methylbuta-1, 3-diene (isoprene) is the monomeric unit of polymer natural rubber.



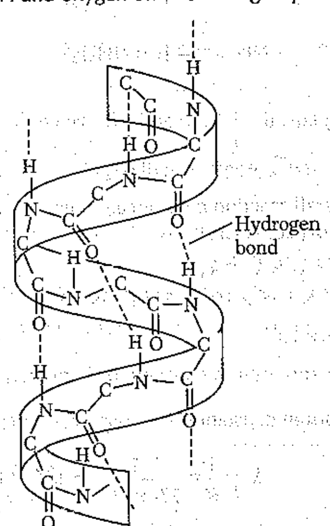
2-methyl buta-1,3-diene (Isoprene) Natural rubber

Value of n is about 10,000.

(d) $CH_2{=}CH{-}CN$ (acrylonitrile) is one of the monomeric unit of polymer buna-N.



19. *(d)* α-helix structure of proteins is stabilised by H-bonds between hydrogen of —NH and oxygen of —C= O group is amino acids.



α-helix

Secondary structure

20. *(b)* (i) In presence of luminous flame, blue cupric metaborate is reduced to colourless cuprous metaborate.

$$2Cu(BO_2)_2 + 2NaBO_2 + C \xrightarrow[\text{flame}]{\text{Luminous}} 2CuBO_2 + Na_2B_4O_7 + CO$$

If non-luminous flame is present in reaction, cupric metaborate is obtained by heating boric anhydride with copper sulphate.

$$CuSO_4 + B_2O_3 \xrightarrow[\text{flame}]{\text{Non-luminous}} \underset{\text{Cupric metaborate}}{Cu(BO)_2} + SO_3$$

So, both statements are false.

21. *(2)* Given, weight of compound $A = 4.5$ g

Molecular weight of compound $A = 90$ g/mol

Volume of solution (in mL) = 250 mL

Now, molarity is defined as number of moles of solute or compound A divided by volume of solution (in L).

$$M = \frac{\text{Number of moles of solute } (n)}{\text{Volume of solution}}$$

$$= \frac{\frac{4.5}{90}}{\frac{250}{1000}} = 0.2 \text{ or } = 2 \times 10^{-1}\text{ M}$$

$$\therefore \quad n = \frac{\text{Weight of solute (compound } A)}{\text{Molecular weight of solute (compound } A)}$$

Hence, $x \times 10^{-1}$ μ

$x = 2$

22. *(8)* Coordination number is the number of nearest neighbours of a central atom in the structure.

bcc has a coordination number of 8 and contains 2 atoms per unit cell.

This is because each atom touches four atoms in the layer above it, four in the layer below it and none in its own layer.

23. *(2)* Given, mass of Li^{3+} = 8.3 times of mass of proton formula,

De-Broglie wavelength, $\lambda = \dfrac{h}{\sqrt{2mqV}}$

Here, h = Planck's constant = 6.624×10^{-34} J-s

m = Mass of atom

q = Charge (or number of electrons)

$$\lambda_{Li} = \frac{h}{\sqrt{2m_{Li}(3e)V}} \quad \text{...(i)}$$

$$\lambda_P = \frac{h}{\sqrt{2m_P(e)V}} \quad \text{...(ii)}$$

Now, Eq. (i) divided by Eq. (ii), we get,

$$\frac{\lambda_{Li}}{\lambda_P} = \sqrt{\frac{m_P(e)V}{m_{Li}(3e)V}}$$

We know that $m_{Li} = 8.3\, m_p$

$$\frac{\lambda_{Li}}{\lambda_P} = \sqrt{\frac{m_P \times e \times V}{8.3\, m_P \times 3e \times V}} = \sqrt{\frac{1}{8.3 \times 3}} = \frac{1}{5} = 0.2$$

$$2 \times 10^{-1}$$

Hence, $x \times 10^{-1}$, $x = 2$

24. *(1380)* For a reaction, $A(g) \longrightarrow B(g)$

Given, K_p (equilibrium constant) = 100

Temperature = 300 K

Pressure = 1 atm

Formula used, $\Delta G^\circ = -RT \ln K_p$ ...(i)

Here, ΔG° = standard Gibb's free energy

R = gas constant = 8.31 J mol^{-1} K^{-1}

Put value in Eq (i), we get

$$\Delta G^\circ = -R(300)\ln 100$$
$$\Delta G^\circ = -R(300)(2)\ln(10)$$

$\because \ln(10) = 2.3$

$$\Delta G^\circ = -R(300)(2)(2.3)$$
$$\Delta G^\circ = -1380R$$

Hence, $\Delta G^\circ = -xR$

$$x = 1380$$

25. *(34.4)* $ClCH_2COOH \rightleftharpoons ClCH_2COO^- + H^+$

$t=0$	1	0	0
at $t=t$	$1-\alpha$	α	α

Here, $i = \dfrac{\text{Final moles}}{\text{Initial moles}}$

$$i = 1-\alpha+\alpha+\alpha \Rightarrow i = 1+\alpha$$

Formula used for freezing point; $\Delta T_f = iK_fm$

Here, K_f = freezing constant of H_2O

$K_f(H_2O) = 1.86\ \text{K kg mol}^{-1}$

m = molarity

$$\Delta T_f = iK_fm$$
$$0.5 = (1+\alpha)(1.86)\frac{(9.45/94.5)}{(500/1000)}$$
$$\frac{5}{3.72} = 1+\alpha \Rightarrow \frac{5}{3.72} - 1 = \alpha$$
$$\Rightarrow \quad \alpha = \frac{1.28}{3.72} = \frac{32}{93} = 0.344$$

Percentage of dissociation = 34.4%

26. *(5)* $Cl_2(g) \rightleftharpoons 2Cl(g)$

Let, moles of both of Cl_2 and Cl molecule be x.

Partial pressure of Cl is, $p_{Cl} = \dfrac{x}{2x}\times 1 = \dfrac{1}{2}$

Partial pressure of Cl_2 is

$$p_{Cl_2} = \frac{x}{2x}\times 1 = \frac{1}{2}$$

Now, $$K_p = \frac{(p_{Cl})^2}{p_{Cl_2}} \Rightarrow K_p = \frac{(1/2)^2}{1/2} = \frac{1}{2} = 0.5$$

or 5×10^{-1}

Hence, $x \times 10^{-1}$

$$x = 5$$

27. *(12)* The two half reaction, one separately are as follows

$S_8 + 16e^- \longrightarrow 8S^{2-}$ (Reduction)

$S_8 + 12H_2O \longrightarrow 4S_2O_3^{2-} + 24H^+ + 16e^-$

$2S_8 + 12H_2O \longrightarrow 8S^{2-} + 4S_2O_3^{2-} + 24H^+$
(Oxidation)

For balancing in basic medium,

Add an equal number of OH^- that of H^+,

we get

$2S_8 + 12H_2O + 24OH^- \longrightarrow 8S^{2-} + 4S_2O_3^{2-} + 24H_2O$

$2S_8 + 24OH^- \longrightarrow 8S^{2-} + 4S_2O_3^{2-} + 12H_2O$

or $S_8 + 12OH^- \longrightarrow 4S^{2-} + 2S_2O_3^{2-} + 6H_2O$ ...(i)

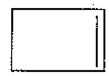
On comparing (i) with

$S_8 + aOH^-(aq) \longrightarrow bS^{2-}(aq) + cS_2O_3^{2-} + dH_2O$

We get, $a = 12$

$b = 4;\ c = 2;\ d = 6$

28. *(26)*



Cyclobutene $\longrightarrow H_2C{=}CH{-}CH{=}CH_2$
1, 3-butadiene

It is a first order isomerisation reaction. Integrated rate law for 1st order reaction is

$$kt = \ln\frac{[A]_0}{[A]_t} \quad \text{...(i)}$$

Here, k = rate constant

$[A]_0$ = initial concentration

$[A]_t$ = concentration at time 't'

Given, $k = 3.3\times 10^{-4}\ s^{-1}$

$[A]_0 = 100 \Rightarrow [A]_t = 100 - 40 = 60$

Put values in Eq. (i), we get

$$3.3\times 10^{-4}s^{-1}\times t = \ln\frac{100}{60}$$
$$t = 1547.95\ s = 25.79\ \text{min} \quad (1\ \text{min} = 60\ s \text{ or } 1\ s = \frac{1}{60}\ \text{min})$$
$$= 26 \text{ minutes}$$

29. *(2)* An amphoteric compound is a molecule or ion that can react with both as an acid or as a base.

BeO = Amphoteric

BaO = Basic

$Be(OH)_2$ = Amphoteric

$Sr(OH)_2$ = Basic

Both beryllium compound BeO and $Be(OH)_2$ are amphoteric in nature while compound BaO and $Sr(OH)_2$ are basic in nature, they form alkaline solution in H_2O.

30. *(1.26)* Given, stability constant value, $k_1 = 10^4$

$$k_2 = 1.58\times 10^3$$
$$k_3 = 5\times 10^2$$
$$k_4 = 10^2$$

$$Cu^{2+} + NH_3 \overset{k_1}{\rightleftharpoons} [Cu(NH_3)]^{2+} \quad \text{...(i)}$$
$$[Cu(NH_3)]^{2+} + NH_3 \overset{k_2}{\rightleftharpoons} [Cu(NH_3)_2]^{2+} \quad \text{...(ii)}$$
$$[Cu(NH_3)_2]^{2+} + NH_3 \overset{k_3}{\rightleftharpoons} [Cu(NH_3)_3]^{2+} \quad \text{...(iii)}$$
$$[Cu(NH_3)_3]^{2+} + NH_3 \overset{k_4}{\rightleftharpoons} [Cu(NH_3)_4]^{2+} \quad \text{...(iv)}$$

⋮ ⋮ ⋮

On adding Eqs. (i), (ii), (iii) and (iv), we get

$$Cu^{2+} + 4NH_3 \overset{k}{\rightleftharpoons} [Cu(NH_3)_4]^{2+}$$

$\therefore$ The overall reaction constant (k) or equilibrium constant for formation of $[Cu(NH_3)_4]^{2+}$ is

$$k = k_1\times k_2\times k_3\times k_4$$
$$k = 10^4\times 1.58\times 10^3\times 5\times 10^2\times 10^2$$
$$k = 7.9\times 10^{11}$$

where, k = equilibrium constant for formation of $[Cu(NH_3)_4]^{2+}$

So, equilibrium constant k' for dissociation of $[Cu(NH_3)_4]^{2+}$ is $\dfrac{1}{k}$.

$$k' = \frac{1}{k} = \frac{1}{7.9\times 10^{11}} = 1.26\times 10^{-12}$$

Hence, $k' = x\times 10^{-12}$

$$x = 1.26$$

MATHEMATICS

1. *(a)* Given, $f(x) = 2x - 1;\ f : R \to R$

$$g(x) = \frac{x - 1/2}{x - 1};\ g : R - \{1\} \to R$$

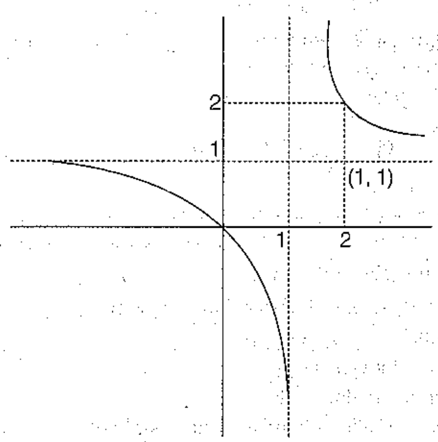
$$f[g(x)] = 2g(x) - 1$$

$$= 2 \times \left(\frac{x - \frac{1}{2}}{x - 1}\right) - 1 = 2 \times \left(\frac{2x - 1}{2(x - 1)}\right) - 1$$

$$= \frac{2x - 1}{x - 1} - 1 = \frac{2x - 1 - x + 1}{x - 1} = \frac{x}{x - 1}$$

$$\therefore \quad f[g(x)] = 1 + \frac{1}{x - 1}$$

Now, draw the graph of $1 + \frac{1}{x - 1}$,



$\because$ Any horizontal line does not cut the graph at more than one points, so it is one-one and here, co-domain and range are not equal, so it is into.

Hence, the required function is one-one into.

2. *(b)* Given, p and q are positive numbers.

$$p + q = 2 \qquad \text{...(i)}$$

$$p^4 + q^4 = 272$$

$$\Rightarrow \quad (p^2 + q^2)^2 - 2p^2q^2 = 272$$

$$\Rightarrow \quad [(p + q)^2 - 2pq]^2 - 2p^2q^2 = 272$$

$$\Rightarrow \quad [(2)^2 - 2pq]^2 - 2p^2q^2 = 272 \qquad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad (4 - 2pq)^2 - 2p^2q^2 = 272$$

$$\Rightarrow 16 + 4p^2q^2 - 16pq - 2p^2q^2 = 272$$

$$\Rightarrow \quad 2p^2q^2 - 16pq - 256 = 0$$

$$\Rightarrow \quad p^2q^2 - 8pq - 128 = 0$$

$$\therefore \quad pq = \frac{8 \pm \sqrt{64 + 4 \times 128}}{2 \times 1}$$

$$= \frac{8 \pm \sqrt{576}}{2}$$

$$= \frac{8 \pm 24}{2}$$

$$\therefore \quad pq = 16, -8$$

Here, $pq = -8$ is not possible as p and q are positive.

$$\therefore \quad pq = 16$$

Now, the equation whose roots are p and q is

$$x^2 - (p + q)x + pq = 0$$

$$\Rightarrow \quad x^2 - 2x + 16 = 0$$

3. *(c)* Given, $3x - 2y - kz = 10$

$$2x - 4y - 2z = 6$$

$$x + 2y - z = 5m$$

Condition for inconsistency $\Rightarrow \Delta = 0$

and atleast one of the $\Delta_x, \Delta_y, \Delta_z$ is non-zero.

$$\text{Now, } \Delta = \begin{vmatrix} 3 & -2 & -k \\ 2 & -4 & -2 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= 3(4 + 4) + 2(-2 + 2) - k(4 + 4)$$

$$= 24 - 8k$$

Now, $\Delta = 0$

$$\therefore \quad 24 - 8k = 0$$

$$\Rightarrow \quad 8k = 24 \Rightarrow k = \frac{24}{8} = 3$$

$$\Delta_x = \begin{vmatrix} 10 & -2 & -k \\ 6 & -4 & -2 \\ 5m & 2 & -1 \end{vmatrix}$$

$$= 10(4 + 4) + 2(-6 + 10m) - 3(12 + 20m)$$

$$= 80 - 12 + 20m - 36 - 60m$$

$$= 32 - 40m = 8(4 - 5m)$$

$$\Delta_y = \begin{vmatrix} 3 & 10 & -k \\ 2 & 6 & -2 \\ 1 & 5m & -1 \end{vmatrix}$$

$$= 3(-6 + 10m) - 10(-2 + 2) - 3(10m - 6)$$

$$= -18 + 30m - 30m + 18$$

$$= 0$$

$$\Delta_z = \begin{vmatrix} 3 & -2 & 10 \\ 2 & -4 & 6 \\ 1 & 2 & 5m \end{vmatrix}$$

$$= 3(-20m - 12) + 2(10m - 6) + 10(4 + 4)$$

$$= -60m - 36 + 20m - 12 + 80$$

$$= -40m + 32 = 8(4 - 5m)$$

Here, either Δ_x or $\Delta_z \neq 0$

$$\Rightarrow \quad 8(4 - 5m) \neq 0$$

$$\Rightarrow \quad m \neq 4/5$$

Hence, the required values are $k = 3;\ m \neq \frac{4}{5}$.

4. *(b)* Given, $(-{}^{15}C_1 + 2\cdot{}^{15}C_2 - 3\cdot{}^{15}C_3 + \dots - 15\cdot{}^{15}C_{15}) + ({}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \dots + {}^{14}C_{11})$

Let $S_1 = -{}^{15}C_1 + 2\cdot{}^{15}C_2 - 3\cdot{}^{15}C_3 + \dots - 15\cdot{}^{15}C_{15}$

$$= \sum_{r=1}^{15}(-1)^r \cdot r \cdot {}^{15}C_r = 15\sum_{r=1}^{15}(-1)^r\, {}^{14}C_{r-1}$$

$$= 15(-{}^{14}C_0 + {}^{14}C_1 - {}^{14}C_2 + \dots - {}^{14}C_{14})$$

$$= 15(0) = 0$$

$$S_2 = {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \dots + {}^{14}C_{11}$$

$$= ({}^{14}C_1 + {}^{14}C_3 + \dots + {}^{14}C_{13}) - {}^{14}C_{13}$$

$$= 2^{13} - 14$$

Now, the required value is

$(-{}^{15}C_1 + 2\cdot{}^{15}C_2 - 3\cdot{}^{15}C_3 + \dots - 15\cdot{}^{15}C_{15}) + ({}^{14}C_1 + {}^{14}C_3 + \dots + {}^{14}C_{11})$

$$= S_1 + S_2 = 0 + 2^{13} - 14 = 2^{13} - 14$$

5. *(a)* Given, $t^2 - 9t + 8 = 0$, is satisfied by

$$e^{(\cos^2 x + \cos^4 x + \cos^6 x + \dots \infty)\log_e 2}$$

Now, $a^{\log_c b} = b^{\log_c a}$

$\therefore$ $e^{(\cos^2 x + \cos^4 x + \dots \infty)\log_e 2}$

$$= 2^{(\cos^2 x + \cos^4 x + \infty)\log_e e} = 2^{\cos^2 x + \cos^4 x + \dots\dots \infty}$$

Here, $\cos^2 x + \cos^4 x + \dots \infty$ are in GP,

where $a = \cos^2 x$

$r = \cos^2 x < 1$ $\quad \left(\because 0 < x < \frac{\pi}{2}\right)$

$\therefore$ $S_\infty = \frac{a}{1-r}$

$\therefore$ $S_\infty = \frac{\cos^2 x}{1-\cos^2 x} = \frac{\cos^2 x}{\sin^2 x} = \cot^2 x$

$\therefore$ $\cos^2 x + \cos^4 x + \dots \infty = \cot^2 x$

Now, $2^{\cos^2 x + \cos^4 x + \dots \infty} = 2^{\cot^2 x}$

Now, roots of equation $t^2 - 9t + 8 = 0$, are

$(t-1)(t-8) = 0$

$t = 1, 8$

$\therefore$ $2^{\cot^2 x} = 1$ or 8

$\Rightarrow$ $2^{\cot^2 x} = 1 = 2^0$ or $2^{\cot^2 x} = 8 = 2^3$

$\Rightarrow$ $\cot^2 x = 0$ or $\cot^2 x = 3 \Rightarrow \cot x = 0$ or $\cot x = \sqrt{3}$

But here, $0 < x < \frac{\pi}{2}$.

$\therefore$ $\cot x = 0$ not possible.

Hence, $\cot x = \sqrt{3}$ is only possible value.

Now, $\frac{2\sin x}{\sin x + \sqrt{3}\cos x}$

Dividing numerator and denominator by $\sin x$, we get

$$\frac{2}{1+\sqrt{3}\cot x}$$

$$= \frac{2}{1+\sqrt{3}\times\sqrt{3}} \quad [\because \cot x = \sqrt{3}]$$

$$= \frac{2}{1+3} = \frac{2}{4} = \frac{1}{2}$$

6. *(a)* Given, $\lim_{x\to 0}\frac{\int_0^{x^2}(\sin\sqrt{t})dt}{x^3}$

$\because$ It is of the form $\frac{0}{0}$.

By differentiating numerator and denominator,

$$\lim_{x\to 0}\frac{\sin\sqrt{x^2}\cdot 2x}{3x^2} = \lim_{x\to 0}\frac{\sin x \cdot 2x}{3x^2}$$

$$= \frac{2}{3}\lim_{x\to 0}\frac{\sin x}{x} = \frac{2}{3}(1) = \frac{2}{3}$$

7. *(a)* Given, $f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x-1)\cos x$

$$f'(x) = \frac{12x^2 - 6x}{6} - 2\cos x + (2x-1)(-\sin x) + \cos x(2)$$

$$= (2x^2 - x) - 2\cos x - 2x\sin x + \sin x + 2\cos x$$

$$= 2x^2 - x - 2x\sin x + \sin x$$

$$= 2x(x - \sin x) - 1(x - \sin x)$$

$f'(x) = (2x-1)(x - \sin x)$

for $x > 0$, $x - \sin x > 0$

$x < 0$, $x - \sin x < 0$

for $x \in (-\infty, 0] \cup \left[\frac{1}{2}, \infty\right)$, $f'(x) \ge 0$

for $x \in \left[0, \frac{1}{2}\right]$, $f'(x) \le 0$

Hence, $f(x)$ increases in $\left[\frac{1}{2}, \infty\right)$.

8. *(b)* Given,

Number of Indians = 6

Number of foreigners = 8

Committee of atleast 2 Indians and double number of foreigners is to be formed. Hence, the required cases are

$$(2I, 4F) + (3I, 6F) + (4I, 8F)$$

$$= {}^6C_2 \times {}^8C_4 + {}^6C_3 \times {}^8C_6 + {}^6C_4 \times {}^8C_8$$

$$= (15 \times 70) + (20 \times 28) + (15 \times 1)$$

$$= 1050 + 560 + 15 = 1625$$

9. *(d)* Given, $f(x) = [x-1]\cos\left(\frac{2x-1}{2}\right)\pi$ where $[\cdot]$ is greatest integer function and $f: R \to R$

$\because$ It is a greatest integer function then we need to check its continuity at $x \in I$ except these it is continuous.

Let, $x = n$ where $n \in I$

Then LHL $= \lim_{x\to n^-}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi$

$$= (n-2)\cos\left(\frac{2n-1}{2}\right)\pi = 0$$

RHL $= \lim_{x\to n^+}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi$

$$= (n-1)\cos\left(\frac{2n-1}{2}\right)\pi = 0$$

and $f(n) = 0$.

Here, $\lim_{x\to n^-} f(x) = \lim_{x\to n^+} f(x) = f(n)$

$\therefore$ It is continuous at every integers.

Therefore, the given function is continuous for all real x.

10. *(b)* Given, $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx$

$$= a \sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$$

Put, $\sin x + \cos x = t$

Also, $\sin^2 x + \cos^2 x + 2 \sin x \cos x = t^2$

$\Rightarrow \quad 2 \sin x \cos x = t^2 - 1$

$\Rightarrow \quad \sin 2x = (t^2 - 1)$

and $(\cos x - \sin x)\, dx = dt$

Now, $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = \int \frac{dx}{\sqrt{8 - (t^2 - 1)}}$

$$= \int \frac{dt}{\sqrt{9 - t^2}} = \sin^{-1}\left(\frac{t}{3}\right) + c$$

Now, according to question,

$$\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a \sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$$

$\Rightarrow \quad \sin^{-1}\left(\frac{t}{3}\right) + c = a \sin^{-1} \frac{(\sin x + \cos x)}{b} + c$

$\Rightarrow 1 \sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + c = a \sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$

$(\because t = \sin x + \cos x)$

$\therefore \quad a = 1$

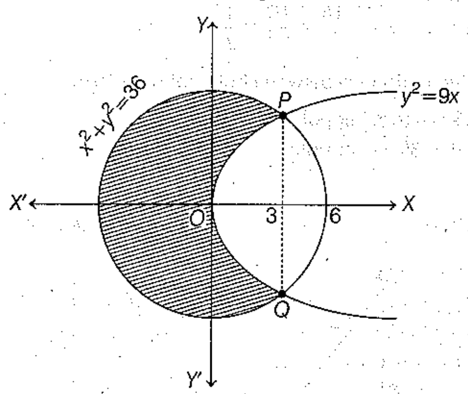
$b = 3$

Hence, $(a, b) = (1, 3)$

11. *(b)* Given, equation of circle $\Rightarrow x^2 + y^2 = 36$

Equation of parabola $\Rightarrow y^2 = 9x$

We have to find area of shaded region.



Required area

$$= \pi r^2 - 2\left[\int_0^3 \sqrt{9x}\, dx + \int_3^6 \sqrt{36 - x^2} dx\right]$$

$$= 36\pi - 12\sqrt{3} - 2 \left| \frac{x}{2} \sqrt{36 - x^2} + 18 \sin^{-1}\left(\frac{x}{6}\right) \right|_3^6$$

$$= 36\pi - 12\sqrt{3} - 2\left[9\pi - \left(\frac{9\sqrt{3}}{2} + 3\pi\right)\right]$$

$$= (24\pi - 3\sqrt{3})$$

12. *(d)* Given, $\frac{dP}{dt} = 0.5P - 450$

and $\quad P(0) = 850$

$\because \quad \frac{dP}{dt} = \frac{1}{2} P - 450 = \frac{P - 900}{2}$

$\Rightarrow \quad \int_0^t \frac{dP}{P - 900} = \int_0^t \frac{dt}{2}$

$\Rightarrow \quad [\log_e |P - 900|]_0^t = \left\{\frac{t}{2}\right\}_0^t$

$\Rightarrow \log_e |P(t) - 900| - \log_e |P(0) - 900| = \frac{t}{2}$

$\Rightarrow \quad \log_e |P(t) - 900| - \log_e 50 = \frac{t}{2}$

Let at $t = t_1$, $P(t) = 0$

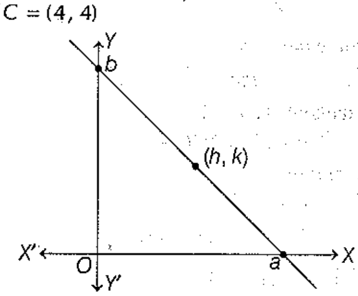
Hence, $\log_e |P(t) - 900| - \log_e 50 = t_1 / 2$

$\therefore \quad t_1 = 2 \log_e 18$

13. *(b)* Given, position of $A = (1, 1)$

Position of $B = (2, 2)$

Position of $C = (4, 4)$



Let x-intercept be a and y-intercept be b.

Equation of line traced is

$$\frac{x}{a} + \frac{y}{b} = 1$$

This is the equation of path, let a point (h, k) lie on this path.

Then $\quad \frac{h}{a} + \frac{k}{b} = 1 \quad$ ...(i)

Also, AM of reciprocal of a and $b = \frac{1}{4}$

$$\therefore \quad \frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{1}{4}$$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{2} \quad \text{... (ii)}$$

On comparing Eqs. (i) and (ii), we get

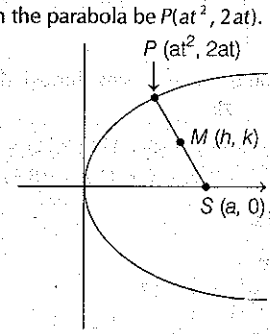
$(h, k) = (2, 2)$

Hence, the required stone is $B(2, 2)$.

14. *(c)* Given, equation of parabola $\Rightarrow y^2 = 4ax$

Focus = $S(a, 0)$

Let any point on the parabola be $P(at^2, 2at)$.



and let the mid-point of PS be $M(h, k)$.

$\therefore \quad h = \frac{at^2 + a}{2}; k = \frac{2at + 0}{2}$

$\Rightarrow \quad t^2 = \frac{2h - a}{a}; t = \frac{k}{a}$

$\Rightarrow \quad t^2 = \frac{k^2}{a^2}$

Now, $\frac{2h - a}{a} = \frac{k^2}{a^2}$

$\Rightarrow \quad 2h - a = \frac{k^2}{a} \Rightarrow k^2 = a(2h - a)$

$\therefore$ Locus of (h, k) is $y^2 = a(2x - a)$

$$y^2 = 2a\left(x - \frac{a}{2}\right)$$

$\therefore$ The directrix of this parabola is

$$x - \frac{a}{2} = -\frac{a}{2} \Rightarrow x = 0$$

15. *(d)* Given, curve $\Rightarrow y = x^3$ ... (i)

$$P(t, t^3)$$

Equation of tangent at $P(t, t^3)$

$$(y - t^3) = 3t^2(x - t) \quad \text{... (ii)}$$

From Eqs. (i) and (ii),

$$x^3 - t^3 = 3t^2(x - t)$$

$\Rightarrow (x - t)(x^2 + t^2 + xt) = 3t^2(x - t)$

$\Rightarrow \quad x^2 + xt - 2t^2 = 0$

$\Rightarrow \quad (x - t)(x + 2t) = 0$

$\Rightarrow \quad x = t$ or $x = -2t$

This is not possible.

Now, the coordinate of $Q = (x, y) = (-2t, (-2t)^3)$ $[\because y = x^3]$

$\therefore \quad Q = (-2t, -8t^3)$

$\therefore$ Ordinate of the point dividing PQ in the ratio 1 : 2 is

$$\frac{2t^3 + (-8t^3)}{1 + 2} = -2t^3$$

16. *(b)* Given, equation of planes are

$$3x + y - 2z = 5$$
$$2x - 5y - z = 7$$

and point $(1, 2, -3)$.

Normal vector of required plane $= \mathbf{n} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix}$

$= \hat{\mathbf{i}}(-1 - 10) - \hat{\mathbf{j}}(-3 + 4) + \hat{\mathbf{k}}(-15 - 2)$

$= -11\hat{\mathbf{i}} - \hat{\mathbf{j}} - 17\hat{\mathbf{k}}$

Now, the equation of plane passing through $(1, 2, -3)$ having normal vector $(-11\hat{\mathbf{i}} - \hat{\mathbf{j}} - 17\hat{\mathbf{k}})$ is

$$-[11(x - 1) + (y - 2) + 17(z + 3)] = 0$$

$\Rightarrow \quad 11x + y + 17z + 38 = 0$

17. *(c)* Given, $P(1, 1, 9)$.

Equation of plane

$$x + y + z = 17$$

Equation of line $\Rightarrow \frac{x - 3}{1} = \frac{y - 4}{2} = \frac{z - 5}{2}$

$\Rightarrow \quad \frac{x - 3}{1} = \frac{y - 4}{2} = \frac{z - 5}{2} = \lambda$ (let)

$\Rightarrow \quad x = \lambda + 3; y = 2\lambda + 4; z = 2\lambda + 5$

$\therefore$ The point we have is $(\lambda + 3, 2\lambda + 4, 2\lambda + 5)$.

$\because$ This point lies on the plane $x + y + z = 17$.

$\therefore \quad \lambda + 3 + 2\lambda + 4 + 2\lambda + 5 = 17$

$\Rightarrow \quad \lambda = 1$

$\therefore$ The coordinate of point is $(4, 6, 7)$

$\therefore$ Required distance between $(1, 1, 9)$ and $(4, 6, 7)$ is

$= \sqrt{(4 - 1)^2 + (6 - 1)^2 + (7 - 9)^2}$

$= \sqrt{9 + 25 + 4} = \sqrt{38}$

18. *(d)* Given, P(odd number 2 times) $= P$(even number 3 times)

$\Rightarrow \quad {}^nC_2\left(\frac{1}{2}\right)^n = {}^nC_3\left(\frac{1}{2}\right)^n$

where, n = Number of times the die is thrown.

$\Rightarrow \quad \frac{n!}{2!(n - 2)!} \times \left(\frac{1}{2}\right)^n = \frac{n!}{3!(n - 3)!} \times \left(\frac{1}{2}\right)^n$

$\Rightarrow \quad \frac{3! \times (n - 3)!}{2! \times (n - 2)!} = 1$

$\Rightarrow \quad \frac{3 \times 2! \times (n - 3)!}{2! \times (n - 2)(n - 3)!} = 1 \Rightarrow 3 = (n - 2)$

$\Rightarrow \quad n = 5$

$\therefore$ Probability of getting an odd number, odd number of times

$= P(1) + P(3) + P(5)$

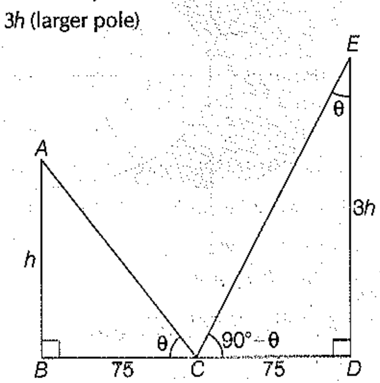
$= {}^5C_1\left(\frac{1}{2}\right)^5 + {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5$

$= \frac{16}{2^5} = \frac{16}{32} = \frac{1}{2}$

19. *(d)* Given, distance between both poles = 150 m

Let $AB = h$ (smaller pole)

and $DE = 3h$ (larger pole)



$\angle ACB = \theta$

Then, $\angle ECD = 90° - \theta$ [$\because$ angles are complementary]

In ΔACB, $\tan\theta = \frac{h}{75}$

Also, in ΔECD, $\tan\theta = \frac{75}{3h}$

$\Rightarrow \quad \frac{h}{75} = \frac{75}{3h}$

$\Rightarrow \quad h^2 = \frac{(75)^2}{3}$

$\therefore \quad h = \sqrt{\frac{75 \times 75}{3}} = 25\sqrt{3}$ m

$\therefore$ Required height = $25\sqrt{3}$ m

20. *(c)* Given, $[A \wedge (A \to B)] \to B$

$= A \wedge (\sim A \vee B) \to B$

$= [(A \wedge \sim A) \vee (A \wedge B)] \to B$

$= (A \wedge B) \to B$

$= \sim A \vee \sim B \vee B$

$= t$

Hence, $[A \wedge (A \to B)] \to B$ is a tautology.

21. *(10)* Given, $\alpha_{\text{least}} = p$

$\alpha_{\max} = q$

Equation given is $z + \alpha | z - 1 | + 2i = 0$; $z \in c$ and $i = \sqrt{-1}$

Let $z = x + iy$

Then, $z + \alpha | z - 1 | + 2i = 0$

$\Rightarrow x + iy + \alpha\sqrt{(x-1)^2 + y^2} + 2i = 0$

$\Rightarrow (x + \alpha\sqrt{(x-1)^2 + y^2}) + i(y + 2) = 0$

$\therefore \quad y + 2 = 0$ and $x + \alpha\sqrt{(x-1)^2 + y^2} = 0$

$y = -2$ and $x^2 = \alpha^2(x^2 + 1 - 2x + y^2)$

$x^2 = \alpha^2(x^2 - 2x + 5) \qquad (\because y = -2)$

$\Rightarrow \quad \alpha^2 = \frac{x^2}{x^2 - 2x + 5}$

$\therefore \quad \alpha^2 \in \left[0, \frac{5}{4}\right]$

$\therefore \quad \alpha \in \left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$

Now, $4(p^2 + q^2) = 4[(\alpha_{\text{least}})^2 + (\alpha_{\max})^2]$

$= 4\left[\left(-\frac{\sqrt{5}}{2}\right)^2 + \left(\frac{\sqrt{5}}{2}\right)^2\right]$

$= 4 \times \left[\frac{5}{4} + \frac{5}{4}\right] = 10$

22. *(6)* Given, B_1, B_2 and B_3 are three independent events.

Let x, y, z be the probability of B_1, B_2, B_3, respectively.

$P(\text{only } B_1 \text{ occur}) = \alpha$

$P(B_1) \cdot P(\overline{B}_2) \cdot P(\overline{B}_3) = \alpha$

$\Rightarrow \quad x \cdot (1 - y) \cdot (1 - z) = \alpha$

$P(\text{only } B_2 \text{ occur}) = \beta$

$P(\overline{B}_1) \cdot P(B_2) \cdot P(\overline{B}_3) = \beta$

$\Rightarrow \quad (1 - x) \cdot y \cdot (1 - z) = \beta$

$P(\text{only } B_3 \text{ occur}) = \gamma$

$\Rightarrow \quad P(\overline{B}_1) \cdot P(\overline{B}_2) \cdot P(B_3) = \gamma$

$\Rightarrow \quad (1 - x) \cdot (1 - y) \cdot z = \gamma$

$P(\text{none occur}) = P$

$P(\overline{B}_1) \cdot P(\overline{B}_2) \cdot P(\overline{B}_3) = P$

$\Rightarrow \quad (1 - x) \cdot (1 - y) \cdot (1 - z) = P$

Now, we have given relations $(\alpha - 2\beta)P = \alpha\beta$

$\Rightarrow [x(1-y)(1-z) - 2y(1-x)(1-z)](1-x)(1-y)(1-z)$

$= x \cdot (1-y)(1-z) \cdot y(1-x)(1-z)$

[putting the value of α, β, P]

$\Rightarrow \quad (1 - z)[x(1 - y) - 2y(1 - x)] = x \cdot y \cdot (1 - z)$

$\Rightarrow \quad x - xy - 2y + 2xy = xy$

$\Rightarrow \quad x = 2y \qquad \ldots \text{(i)}$

Similarly, on solving the second relation, $(\beta - 3\gamma)P = 2\beta\gamma$ by putting β, γ and P,

we get $y = 3z \qquad \ldots \text{(ii)}$

From Eqs. (i) and (ii), we get

$x = 2 \times 3z$

$\Rightarrow \quad x = 6z \Rightarrow \frac{x}{z} = 6$

Now, $\frac{P(B_1)}{P(B_3)} = \frac{x}{z} = 6$

$\therefore$ Required answer is 6.

23. *(17)* Given, $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$

$\Rightarrow \quad |P| = \begin{vmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{vmatrix} = (20 + 12\alpha)$

According to the question,

$PQ = kI_3 \Rightarrow Q = kP^{-1}I_3$

Now, $Q = \frac{k}{|P|}(\text{adj } P)\, I_3$

$= \frac{k}{(20 + 12\alpha)} \begin{bmatrix} 5\alpha & -10 & -\alpha \\ 3\alpha & 6 & -3\alpha - 4 \\ -10 & 12 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\because \quad q_{23} = -\frac{k}{8}$

$\therefore \quad \frac{k}{(20 + 12\alpha)}(-3\alpha - 4) = -\frac{k}{8}$

$\Rightarrow \quad 2(3\alpha + 4) = 5 + 3\alpha \Rightarrow 3\alpha = -3$

$\Rightarrow \quad \alpha = -1$

Also, $|Q| = \frac{k^3 |I|}{|P|}$

$\Rightarrow \quad \frac{k^2}{2} = \frac{k^3}{20 + 12\alpha} \Rightarrow 20 + 12\alpha = 2k$

$\Rightarrow \quad 2k = 20 - 12 \qquad [\because \alpha = -1]$

$\Rightarrow \quad 2k = 8$

$k = 4$

$\therefore$ Required value of $k^2 + \alpha^2 = 4^2 + (-1)^2 = 17$

24. *(540)* Given, M is a 3×3 matrix.

Let $M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, then $M^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

Now, $M^T M = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} \times \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

Sum of diagonal elements

$= a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$ ...(i)

According to the question, the entries are {0, 1, 2}. [$\because M^T M = 7$]

i.e. $\{a, b, c, \ldots, h, i\} = \{0, 1, 2\}$

So, for Eq. (i) to be true, there are two cases.

Case I When 7 – 1's are there and 2 – 0's are there.

$\Rightarrow \quad {}^9C_7 \times {}^2C_2 = 36$ ways of arrangements.

Case II When 1 – 2 is there, 3 – 1's and 5 – 0's are there.

$${}^9C_1 \times {}^8C_3 \times {}^5C_5 = 9 \times \frac{8!}{3!5!} \times 1$$

$= 504$ ways of arrangements.

$\therefore$ Total possible arrangements $= 36 + 504 = 540$

25. *(5)* Given, $A = \{n \in N : n \text{ is a 3-digit number}\}$

$B = \{9k + 2 : k \in N\}$

$c = \{9k + l : k \in N\}$

$\because$ 3 digit number of the form $3k + 2$ are

{101, 109, ... 992}

$$\Rightarrow \quad \text{Sum} = \frac{100}{2}[101 + 992] = \frac{100 \times 1093}{2}$$

Similarly, 3-digit number of the form $9k + 5$ is

$$\frac{100}{2}[104 + 995] = \frac{100 \times 1099}{2}$$

[$\because$ numbers are 104, 113, ..., 995]

$$\text{Their sum} = \frac{100 \times 1093}{2} + \frac{100 \times 1099}{2}$$

$= 100 \times 1096 = 400 \times 274$

Hence, we can say the value of $l = 5$

as the second series of numbers obtained by set C is of the form $9k + 5$.

$\therefore$ Required value of $l = 5$

26. *(9)* Given, equation $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = \alpha$

Let $f(x) = \frac{4}{\sin x} + \frac{1}{1 - \sin x}$

$$\Rightarrow \quad y = \frac{4 - 3\sin x}{\sin x(1 - \sin x)}$$

Let $\sin x = t$ when $t \in (0, 1)$ as $x \in \left(0, \frac{\pi}{2}\right)$. $\left[\begin{array}{l} \because t_1 = \sin 0 = 0 \\ t_2 = \sin \frac{\pi}{2} = 1 \end{array}\right]$

Now, $y = \frac{4 - 3t}{t - t^2}$

$$\frac{dy}{dt} = \frac{-3(t - t^2) - (1 - 2t)(4 - 3t)}{(t - t^2)^2} = 0$$

$\Rightarrow 3t^2 - 3t - (4 - 11t + 6t^2) = 0$

$\Rightarrow \quad 3t^2 - 8t + 4 = 0$

$\Rightarrow \quad 3t^2 - 6t - 2t + 4 = 0$

$\Rightarrow \quad 3t(t - 2) - 2(t - 2) = 0$

$\Rightarrow \quad (t - 2)(3t - 2) = 0$

$\therefore \quad t - 2 = 0$ or $3t - 2 = 0$

$t = 2$ or $t = 2/3$

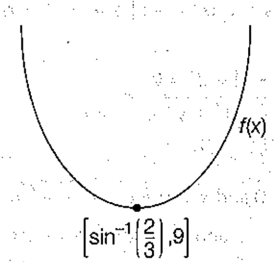
$\sin x = 2$ or $\sin x = \frac{2}{3}$

$x = \sin^{-1}\left(\frac{2}{3}\right)$ [$\because \sin\theta \le 1$]

Also, $y = \frac{4 - 3t}{t - t^2} = \frac{4 - 3 \times 2/3}{\frac{2}{3} - \left(\frac{2}{3}\right)^2}$ [$\because t = 2/3$]

$y = 9$

Now, required graph is



From the graph, minimum value of

$f(x) \ge 9$

$\alpha \ge 9$

$\therefore \alpha$ has minimum value 9.

27. *(3)* Given, $\int_{-a}^{a} (|x| + |x - 2|)\, dx = 22$

$\Rightarrow \int_{-a}^{0} (-2x + 2)\, dx + \int_{0}^{2} (x + 2 - x)\, dx + \int_{2}^{a} (2x - 2)\, dx = 22$

$\Rightarrow (x^2 - 2x)\Big|_0^{-a} + (2x)\Big|_0^2 + (x^2 - 2x)\Big|_2^a = 22$

$\Rightarrow \quad a^2 + 2a + 4 + a^2 - 2a - (4 - 4) = 22$

$\Rightarrow \quad 2a^2 = 18$

$\Rightarrow \quad a^2 = 9$

$\Rightarrow \quad a = 3$

$\therefore \quad \int_a^{-a} x + [x]\, dx = \int_3^{-3} (x + [x])\, dx$

$= -\int_{-3}^{3} (x + [x])\, dx$

$= -[-3 - 2 - 1 + 1 + 2]$

$= -[-3] = 3$

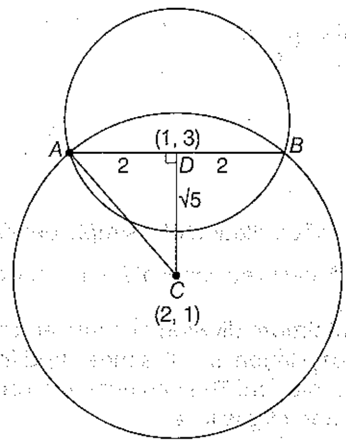
$\therefore$ Required value is 3.

28. *(3)* Given, circle $\Rightarrow x^2 + y^2 - 2x - 6y + 6 = 0$

Coordinate of $D = (1, 3)$

Radius $= r = \sqrt{1^2 + 3^2 - 6}$

$= \sqrt{4} = 2$ units



$CD = \sqrt{(2-1)^2 + (1-3)^2}$

$= \sqrt{1+4} = \sqrt{5}$

Now, by using Pythagoras theorem in ΔADC,

$AC^2 = AD^2 + CD^2$

$= (2)^2 + (\sqrt{5})^2$

$= 4 + 5$

$AC^2 = 9$

$AC = \sqrt{9} = 3$

$\therefore$ Required radius = 3

29. *(75)* Given, $\mathbf{c}$ is co-planar with $\mathbf{a}$ and $\mathbf{b}$

$\mathbf{a} \cdot \mathbf{c} = 7$

$\mathbf{b} \perp \mathbf{c} \Rightarrow \mathbf{b} \cdot \mathbf{c} = 0$

$\mathbf{a} = -\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

$\mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{k}}$

Now, $\mathbf{c} = \lambda[\mathbf{b} \times (\mathbf{a} \times \mathbf{b})]$ [$\because$ $\mathbf{c}$ is coplanar with $\mathbf{a}$ and $\mathbf{b}$]

$= \lambda[(\mathbf{b} \cdot \mathbf{b})\, \mathbf{a} - (\mathbf{b} \cdot \mathbf{a})\, \mathbf{b}]$

$= \lambda[(\sqrt{5})^2(-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - (-2+1)(2\hat{\mathbf{i}} + \hat{\mathbf{k}})]$

$= \lambda[5(-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + 2\hat{\mathbf{i}} + \hat{\mathbf{k}}]$

$= \lambda(-3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$

Now, $\mathbf{c} \cdot \mathbf{a} = 7$

$\Rightarrow \lambda(-3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 6\hat{\mathbf{k}}) \cdot (-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 7$

$\Rightarrow 3\lambda + 5\lambda + 6\lambda = 7$

$\Rightarrow \lambda = \frac{7}{14} = \frac{1}{2}$

$\therefore 2\,|\mathbf{a} + \mathbf{b} + \mathbf{c}|^2$

$= 2\left|\left(\frac{-3}{2} - 1 + 2\right)\hat{\mathbf{i}} + \left(\frac{5}{2} + 1\right)\hat{\mathbf{j}} + (3 + 1 + 1)\hat{\mathbf{k}}\right|^2$ [by putting a, b, c]

$= 2\left(\frac{1}{4} + \frac{49}{4} + 25\right) = 25 + 50 = 75$

$\therefore$ Required value is 75.

30. *(1)* Given, $\lim_{n \to \infty} \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1 + r + r^2}\right)\right\}$

$= \tan\left(\lim_{n \to \infty} \sum_{r=1}^{n} [\tan^{-1}(r+1) - \tan^{-1} r]\right)$

$= \tan\left(\lim_{n \to \infty}\left(\tan^{-1}(n+1) - \frac{\pi}{4}\right)\right)$

$= \tan\left(\frac{\pi}{2} - \frac{\pi}{4}\right)$

$= \tan\frac{\pi}{4} = 1$

Hence, the required value is 1.

JEE Main 2021

24 FEBRUARY SHIFT II

PHYSICS

Section A : Objective Type Questions

1. When a particle executes SHM, the nature of graphical representation of velocity as a function of displacement is

a. circular **b.** elliptical **c.** parabolic **d.** straight line

2. Two electrons each are fixed at a distance $2d$. A third charge proton placed at the mid-point is displaced slightly by a distance $x(x << d)$ perpendicular to the line joining the two fixed charges. Proton will execute simple harmonic motion having angular frequency? (m = mass of charged particle)

a. $\left(\frac{2q^2}{\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$ **b.** $\left(\frac{\pi\varepsilon_0 md^3}{2q^2}\right)^{\frac{1}{2}}$ **c.** $\left(\frac{q^2}{2\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$ **d.** $\left(\frac{2\pi\varepsilon_0 md^3}{q^2}\right)^{\frac{1}{2}}$

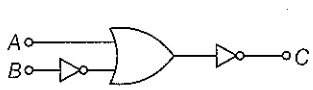
3. On the basis of kinetic theory of gases, the gas exerts pressure because its molecules

a. continuously lose their energy till it reaches wall
b. are attracted by the walls of container
c. continuously stick to the walls of container
d. suffer change in momentum when impinge on the walls of container

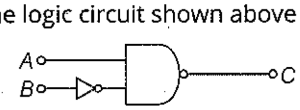
4. A soft ferromagnetic material is placed in an external magnetic field. The magnetic domains

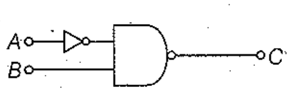
a. increase in size but no change in orientation
b. have no relation with external magnetic field
c. decrease in size and changes orientation
d. may increase or decrease in size and change its orientation

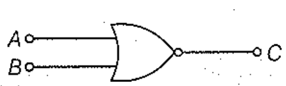
5.

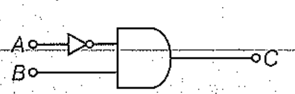


The logic circuit shown above is equivalent to

a.


b.


c.


d.


6. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 1.0 m from metre scale having a minimum division of 1 mm and time of one complete oscillation is 1.95 s measured from stopwatch of 0.01 s resolution. The percentage error in the determination of g will be

a. 1.13% **b.** 1.03% **c.** 1.33% **d.** 1.30%

7. Given below are two statements:

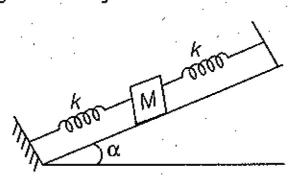
Statement I p-n junction diodes can be used to function as transistor, simply by connecting two diodes, back to back, which acts as the base terminal.

Statement II In the study of transistor, the amplification factor β indicates ratio of the collector current to the base current.

In the light of the above statements, choose the correct answer from the options given below.

a. Statement I is false but Statement II is true.
b. Both Statement I and Statement II are true.
c. Both Statement I and Statement II are false.
d. Statement I is true but Statement II is false.

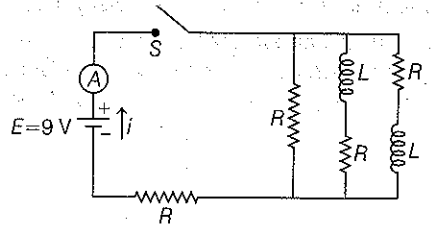
8. In the given figure, a body of mass M is held between two massless springs, on a smooth inclined plane. The free ends of the springs are attached to firm supports. If each spring has spring constant k, then the frequency of oscillation of given body is



a. $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$ **b.** $\frac{1}{2\pi}\sqrt{\frac{2k}{Mg\sin\alpha}}$

c. $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ **d.** $\frac{1}{2\pi}\sqrt{\frac{k}{Mg\sin\alpha}}$

9. Figure shows a circuit that contains four identical resistors with resistance $R = 2.0\ \Omega$, two identical inductors with inductance $L = 2.0$ mH and an ideal battery with electromotive force $E = 9$ V. The current i just after the switch S is closed will be

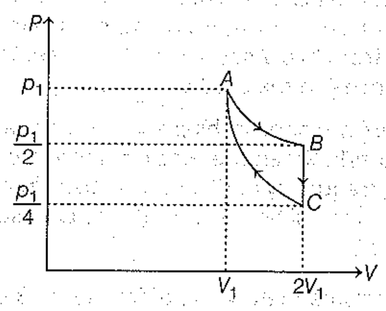


a. 2.25 A **b.** 3.0 A **c.** 3.37 A **d.** 9 A

10. The de-Broglie wavelength of a proton and α-particle are equal. The ratio of their velocities is

a. 4 : 3 **b.** 4 : 1 **c.** 4 : 2 **d.** 1 : 4

11. If one mole of an ideal gas at (p_1, V_1) is allowed to expand reversibly and isothermally (A to B), its pressure is reduced to one-half of the original pressure (see figure). This is followed by a constant volume cooling till its pressure is reduced to one-fourth of the initial value ($B \to C$). Then, it is restored to its initial state by a reversible adiabatic compression (C to A). The net work done by the gas is equal to



a. $RT\left(\ln 2 - \frac{1}{2(\gamma - 1)}\right)$ **b.** $-\frac{RT}{2(\gamma - 1)}$

c. 0 **d.** $RT \ln 2$

12. An X-ray tube is operated at 1.24 million volt. The shortest wavelength of the produced photon will be

a. 10^{-3} nm **b.** 10^{-1} nm **c.** 10^{-2} nm **d.** 10^{-4} nm

13. Which of the following equations represents a travelling wave?

a. $y = A\sin(15x - 2t)$ **b.** $y = Ae^{-x^2}(vt + \theta)$

c. $y = Ae^{x}\cos(\omega t - \theta)$ **d.** $y = A\sin x \cos \omega t$

14. According to Bohr atom model, in which of the following transitions will the frequency be maximum ?

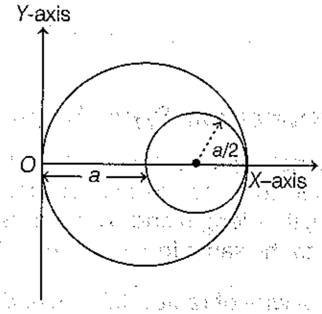
a. $n = 4$ to $n = 3$ **b.** $n = 2$ to $n = 1$

c. $n = 5$ to $n = 4$ **d.** $n = 3$ to $n = 2$

15. If the source of light used in a Young's double slit experiment is changed from red to violet, then

a. the consecutive fringe lines will come closer
b. the central bright fringe will become a dark fringe
c. the fringes will become brighter
d. the intensity of minima will increase

16. A circular hole of radius $\left(\frac{a}{2}\right)$ is cut out of a circular disc of radius a as shown in figure. The centroid of the remaining circular portion with respect to point O will be



a. $\frac{1}{6}a$ **b.** $\frac{10}{11}a$

c. $\frac{5}{6}a$ **d.** $\frac{2}{3}a$

17. Zener breakdown occurs in a p-n junction having p and n both

a. lightly doped and have wide depletion layer
b. heavily doped and have narrow depletion layer
c. lightly doped and have narrow depletion layer
d. heavily doped and have wide depletion layer

18. Match List-I with List-II.

List-I		List-II
A. Source of microwave frequency	1.	Radioactive decay of nucleus
B. Source of infrared frequency	2.	Magnetron
C. Source of gamma rays	3.	Inner shell electrons
D. Source of X-rays	4.	Vibration of atoms and molecules
	5.	LASER
	6.	R-C circuit

Choose the correct answer from the options given below.

	A	B	C	D		A	B	C	D
a.	6	4	1	5	**b.**	6	5	1	4
c.	2	4	6	3	**d.**	2	4	1	3

19. A particle is projected with velocity v_0 along X-axis. A damping force is acting on the particle which is proportional to the square of the distance from the origin, i.e. $ma = -\alpha x^2$. The distance at which the particle stops is

a. $\left(\frac{3mv_0^2}{2\alpha}\right)^{\frac{1}{2}}$ **b.** $\left(\frac{2mv_0}{3\alpha}\right)^{\frac{1}{3}}$

c. $\left(\frac{2mv_0^2}{3\alpha}\right)^{\frac{1}{2}}$ **d.** $\left(\frac{3mv_0^2}{2\alpha}\right)^{\frac{1}{3}}$

20. A body weights 49 N on a spring balance at the North pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator? (Use, $g = \frac{GM}{R^2} = 9.8\ ms^{-2}$ and radius of earth, $R = 6400$ km)

a. 49 N
b. 48.83 N
c. 49.83 N
d. 49.17 N

Section B : Numerical Type Questions

21. A uniform metallic wire is elongated by 0.04 m when subjected to a linear force F. The elongation, if its length and diameter is doubled and subjected to the same force will be .... cm.

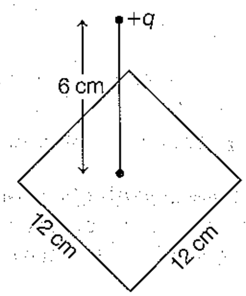
22. A cylindrical wire of radius 0.5 mm and conductivity 5×10^7 S/m is subjected to an electric field of 10 mV/m. The expected value of current in the wire will be $x^3\pi$ mA. The value of x is ....

23. A uniform thin bar of mass 6 kg and length 2.4 m is bent to make an equilateral hexagon. The moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is .... $\times 10^{-1}$ kg-m^2.

24. Two solids A and B of mass 1 kg and 2 kg respectively are moving with equal linear momentum. The ratio of their kinetic energies $(KE)_A : (KE)_B$ will be $\frac{A}{1}$, so the value of A will be ....

25. The root mean square speed of molecules of a given mass of a gas at 27°C and 1 atmosphere pressure is 200 ms^{-1}. The root mean square speed of molecules of the gas at 127°C and 2 atmosphere pressure is $\frac{x}{\sqrt{3}}$ ms^{-1}. The value of x will be ....

26. A point charge of + 12 μC is at a distance 6 cm vertically above the centre of a square of side 12 cm as shown in figure. The magnitude of the electric flux through the square will be .... $\times 10^3$ N-m^2/C.



27. A signal of 0.1 kW is transmitted in a cable. The attenuation of cable is − 5 dB per km and cable length is 20 km. The power received at receiver is 10^{-x} W. The value of x is ....
[Gain in dB $= 10 \log_{10}\left(\frac{P_0}{P_i}\right)$]

28. A series L-C-R circuit is designed to resonate at an angular frequency $\omega_0 = 10^5$ rad/s. The circuit draws 16 W power from 120 V source at resonance. The value of resistance R in the circuit is .... Ω.

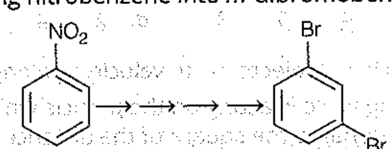
29. Two cars are approaching each other at an equal speed of 7.2 km/h. When they see each other, both blow horns having frequency of 676 Hz. The beat frequency heard by each driver will be .... Hz. [Velocity of sound in air is 340 m/s.]

30. An electromagnetic wave of frequency 3 GHz enters a dielectric medium of relative electric permittivity 2.25 from vacuum. The wavelength of this wave in that medium will be .... $\times 10^{-2}$ cm.

CHEMISTRY

Section A : Objective Type Questions

1. What is the correct sequence of reagents used for converting nitrobenzene into *m*-dibromobenzene?

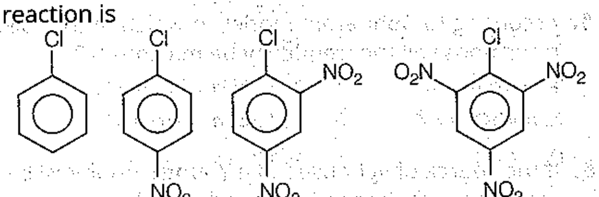


a. $\xrightarrow{NaNO_2}$ / $\xrightarrow{HCl}$ / $\xrightarrow{KBr}$ / $\xrightarrow{H^+}$
b. $\xrightarrow{Br_2/Fe}$ / $\xrightarrow{Sn/HCl}$ / $\xrightarrow{NaNO_2/HCl}$ / $\xrightarrow{CuBr/HBr}$
c. $\xrightarrow{Sn/HCl}$ / $\xrightarrow{KBr}$ / $\xrightarrow{Br_2}$ / $\xrightarrow{H^+}$
d. $\xrightarrow{Sn/HCl}$ / $\xrightarrow{Br_2}$ / $\xrightarrow{NaNO_2}$ / $\xrightarrow{NaBr}$

2. Most suitable salt which can be used for efficient clotting of blood will be

a. $NaHCO_3$
b. $FeSO_4$
c. $Mg(HCO_3)_2$
d. $FeCl_3$

3. The correct order of the following compounds showing increasing tendency towards nucleophilic substitution reaction is



(i) (ii) (iii) (iv)

a. (iv) < (iii) < (ii) < (i) **b.** (iv) < (i) < (ii) < (iii)
c. (iv) < (i) < (iii) < (ii) **d.** (i) < (ii) < (iii) < (iv)

4. According to Bohr's atomic theory,

I. kinetic energy of electron is $\propto \frac{Z^2}{n^2}$

II. the product of velocity (v) of electron and principal quantum number (n), $vn \propto Z^2$.

III. frequency of revolution of electron in an orbit is $\propto \frac{Z^3}{n^3}$

IV. coulombic force of attraction on the electron is $\propto \frac{Z^3}{n^4}$

Choose the most appropriate answer from the options given below.

a. Only III **b.** Only I **c.** I, III and IV **d.** I and IV

5. Match List-I with List-II.

	List-I	List-II
A.	$R—\overset{O}{\overset{\|\|}{C}}—Cl \longrightarrow R—CHO$	1. Br_2/ NaOH
B.	$R—CH_2—COOH \longrightarrow R—\underset{Cl}{\underset{\|}{C}H}—COOH$	2. H_2/ Pd - $BaSO_4$
C.	$R—\overset{O}{\overset{\|\|}{C}}—NH_2 \longrightarrow R—NH_2$	3. Zn(Hg) / Conc.HCl
D.	$R—\overset{O}{\overset{\|\|}{C}}—CH_3 \longrightarrow R—CH_2—CH_3$	4. Cl_2 / Red P, H_2O

Choose the correct answer from the options given below.

	A	B	C	D		A	B	C	D
a.	2	1	4	3	**b.**	3	4	1	2
c.	2	4	1	3	**d.**	3	1	4	2

6. The calculated magnetic moments (spin only value) for species $[FeCl_4]^{2-}$, $[Co(C_2O_4)_3]^{3-}$ and MnO_4^{2-} respectively are

a. 5.82, 0 and 0 BM **b.** 4.90, 0 and 1.73 BM
c. 5.92, 4.90 and 0 BM **d.** 4.90, 0 and 2.83 BM

7. Match List-I with List-II.

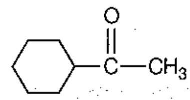
	List-I (Salt)		List-II (Flame colour wavelength)
A.	LiCl	1.	455.5 nm
B.	NaCl	2.	670.8 nm
C.	RbCl	3.	780.0 nm
D.	CsCl	4.	589.2 nm

Choose the correct answer from the options given below.

	A	B	C	D		A	B	C	D
a.	4	2	3	1	**b.**	2	1	4	3
c.	1	4	2	3	**d.**	2	4	3	1

8. Which one of the following carbonyl compounds cannot be prepared by addition of water on an alkyne in the presence of $HgSO_4$ and H_2SO_4?

a. $CH_3—\overset{O}{\overset{\|\|}{C}}—H$

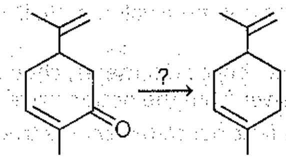
b. 

c. $CH_3—CH_2—\overset{O}{\overset{\|\|}{C}}—H$

d. $CH_3—\overset{O}{\overset{\|\|}{C}}—CH_2CH_3$

9. In polymer buna-S: 'S' stands for

a. sulphonation **b.** strength
c. sulphur **d.** styrene

10. 

Which of the following reagent is suitable for the preparation of the product in the above reaction?

a. $NaBH_4$ **b.** $NH_2—NH_2 / C_2H_5\overset{\ominus}{O}\overset{\oplus}{Na}$
c. Ni / H_2 **d.** Red P + Cl_2

11. Match List-I and List-II.

	List-I		List-II
A.	Valium	1.	Antifertility drug
B.	Morphine	2.	Pernicious anaemia
C.	Norethindrone	3.	Analgesic
D.	Vitamin B_{12}	4.	Tranquiliser

Choose the correct answer from the option given below.

	A	B	C	D		A	B	C	D
a.	4	3	2	1	**b.**	4	3	1	2
c.	2	4	3	1	**d.**	1	3	4	2

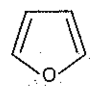
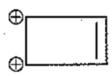
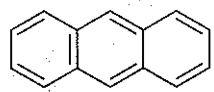
12. Match List-I with List-II.

	List I (Metal)		List II (Ores)
A.	Aluminium	1.	Siderite
B.	Iron	2.	Calamine
C.	Copper	3.	Kaolinite
D.	Zinc	4.	Malachite

Choose the correct answer from the options given below.

	A	B	C	D		A	B	C	D
a.	4	3	2	1	b.	2	4	1	3
c.	1	2	3	4	d.	3	1	4	2

13. Which one of the following compounds is non-aromatic ?

a. 
b. 
c. 
d. 

14. What is the correct order of the following elements with respect to their density ?

a. Cr < Zn < Co < Cu < Fe
b. Zn < Cu < Co < Fe < Cr
c. Zn < Cr < Fe < Co < Cu
d. Cr < Fe < Co < Cu < Zn

15. Given below are two statements.

Statement I The value of the parameter "Biochemical Oxygen Demand (BOD)" is important for survival of aquatic life.

Statement II The optimum value of BOD is 6.5 ppm.

In the light of the above statements, choose the most appropriate answer from the options given below.

a. Statement I is false but statement II is true.
b. Both statements are true.
c. Statement I is true but statement II is false.
d. Both statements are false.

16. The incorrect statement among the following is

a. $VOSO_4$ is a reducing agent.
b. Cr_2O_3 is an amphoteric oxide.
c. RuO_4 is an oxidising agent.
d. Red colour of ruby is due to the presence of Co^{3+}.

17. The correct shape and I—I—I bond angles respectively in I_3^- ion are

a. distorted trigonal planar, 135° and 90°
b. T-shaped, 180° and 90°
c. Trigonal planar, 120°
d. Linear, 180°

18. Given below are two statements.

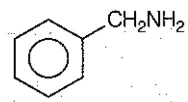
One is labelled as Assertion A and the other is labelled as Reason R.

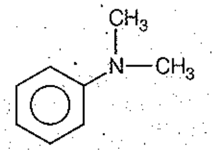
Assertion A Hydrogen is the most abundant element in the universe, but it is not the most abundant gas in the troposphere.

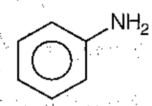
Reason R Hydrogen is the lightest element. In the light of the above statements, choose the correct answer from the options given below.

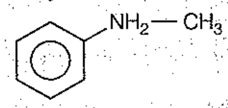
a. A is true but R is false.
b. Both A and R are true and R is the correct explanation of A.
c. A is false but R is true.
d. Both A and R are true but R is not the correct explanation of A.

19. The diazonium salt of which of the following compounds will form a coloured dye on reaction with β-naphthol in NaOH ?

a. 

b. 

c. 

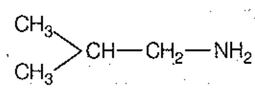
d. 

20. The correct set from the following in which both pairs are in correct order of melting point is

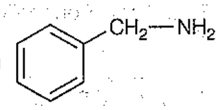
a. LiF > LiCl, MgO > NaCl
b. LiCl > LiF, NaCl > MgO
c. LiF > LiCl, NaCl > MgO
d. LiCl > LiF, MgO > NaCl

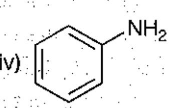
Section B : Numerical Type Questions

21. The total number of amines among the following which can be synthesised by Gabriel synthesis is ....

(i) 

(ii) $CH_3CH_2NH_2$

(iii) 

(iv) 

22. Among the following allotropic forms of sulphur, the number of allotropic forms, which will show paramagnetism is ....

(i) α-sulphur (ii) β-sulphur (iii) S_2-form

23. The formula of a gaseous hydrocarbon, which requires 6 times of its own volume of O_2 for complete oxidation and produces 4 times its own volume of CO_2 is C_xH_y. The value of y is ....

24. The volume occupied by 4.75 g of acetylene gas at 50°C and 740 mm Hg pressure is ......L (Rounded off to the nearest integer).
[Given, $R = 0.0826$ L atm K^{-1} mol^{-1}]

25. C_6H_6 freezes at 5.5°C. The temperature at which a solution 10 g of C_4H_{10} in 200 g of C_6H_6 freeze is ....°C. (The molal freezing point depression constant of C_6H_6 is 5.12°C/m.)

26. The magnitude of the change in oxidising power of the MnO_4^- / Mn^{2+} couple is $x \times 10^{-4}$ V, if the H^+ concentration is decreased from 1 M to 10^{-4} M at 25°C. (Assume concentration of MnO_4^- and Mn^{2+} to be same on change in H^+ concentration). The value of x is .... (Rounded off to the nearest integer).

$$\left[\text{Given, } \frac{2.303\,RT}{F} = 0.059\right]$$

27. The solubility product of PbI_2 is 8.0×10^{-9}. The solubility of lead iodide in 0.1 molar solution of lead nitrate is $x \times 10^{-6}$ mol/L. The value of x is .... (Rounded off to the nearest integer).
[Given, : $\sqrt{2} = 1.41$]

28. Sucrose hydrolyses in acid solution into glucose and fructose following first order rate law with a half-life of 3.33 h at 25°C. After 9 h, the fraction of sucrose remaining is f. The value of $\log_{10}(1/f)$ is ______ $\times 10^{-2}$ (Rounded off to the nearest integer).
[Assume, $\ln 10 = 2.303$, $\ln 2 = 0.693$]

29. 1.86 g of aniline completely reacts to form acetanilide. 10% of the product is lost during purification. Amount of acetanilide obtained after purification (in g) is ______ $\times 10^{-2}$.

30. Assuming ideal behaviour, the magnitude of $\log K$ for the following reaction at 25°C is $x \times 10^{-1}$. The value of x is .... (Integer answer)

$3HC \equiv CH(g) \rightleftharpoons C_6H_6(l)$

[Given, $\Delta_f G^\circ(HC \equiv CH) = -2.04 \times 10^5$ J mol^{-1}, $\Delta_f G^\circ(C_6H_6) = -1.24 \times 10^5$ J mol^{-1}, $R = 8.314$ J K^{-1} mol^{-1}]

MATHEMATICS

Section A : Objective Type Questions

1. For the statements p and q, consider the following compound statements

A. $[\sim q \wedge (p \to q)] \to \sim p$

B. $[(p \vee q) \wedge \sim p] \to q$

Then, which of the following statement(s) is/are correct?

a. (A) and (B) both are not tautologies.
b. (A) and (B) both are tautologies.
c. (A) is a tautology but not (B).
d. (B) is a tautology but not (A).

2. Let $a, b \in R$. If the mirror image of the point $P(a, 6, 9)$ with respect to the line $\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$ is $(20, b, -a-9)$, then $|a + b|$ is equal to

a. 88 **b.** 86
c. 84 **d.** 90

3. The vector equation of the plane passing through the intersection of the planes $\mathbf{r} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 1$ and $\mathbf{r} \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) = -2$ and the point $(1, 0, 2)$ is

a. $\mathbf{r} \cdot (\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = \frac{7}{3}$ **b.** $\mathbf{r} \cdot (3\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = 7$
c. $\mathbf{r} \cdot (\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = 7$ **d.** $\mathbf{r} \cdot (\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = \frac{7}{3}$

4. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the coordinates of P are

a. (3, 13) **b.** (1, 5) **c.** (−2, 8) **d.** (2, 8)

5. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 s at the speed of 432 km/h, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is

a. $1800\sqrt{3}$ m **b.** $3600\sqrt{3}$ m
c. $2400\sqrt{3}$ m **d.** $1200\sqrt{3}$ m

6. If $n \geq 2$ is a positive integer, then the sum of the series ${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + {}^4C_2 + \dots + {}^nC_2)$ is

a. $\frac{n(n-1)(2n+1)}{6}$
b. $\frac{n(n+1)(2n+1)}{6}$
c. $\frac{n(2n+1)(3n+1)}{6}$
d. $\frac{n(n+1)^2(n+2)}{12}$

7. Let $f: R \to R$ be defined as

$$f(x) = \begin{cases} -55x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \leq x < 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x \geq 4 \end{cases}$$

Let $A = \{x \in R : f \text{ is increasing}\}$. Then, A is equal to

a. $(-\infty, -5) \cup (4, \infty)$
b. $(-5, \infty)$
c. $(-\infty, -5) \cup (-4, \infty)$
d. $(-5, -4) \cup (4, \infty)$

8. Let f be a twice differentiable function defined on R, such that $f(0) = 1$, $f'(0) = 2$ and $f''(x) \neq 0$ for all $x \in R$.

If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all $x \in R$, then the value of $f(1)$ lies in the interval

a. (9, 12) **b.** (6, 9)
c. (0, 3) **d.** (3, 6)

9. For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$?

a. $x^2 + y^2 = 7$ **b.** $y^2 = \frac{1}{6\sqrt{3}}x$
c. $2x^2 - 18y^2 = 9$ **d.** $x^2 + 9y^2 = 9$

10. The value of the integral $\int_1^3 [x^2 - 2x - 2]\, dx$, where $[x]$ denotes the greatest integer less than or equal to x, is

a. $-\sqrt{2}-\sqrt{3}+1$ **b.** $-\sqrt{2}-\sqrt{3}-1$
c. -5 **d.** -4

11. A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is

a. $1/\sqrt{7}$ **b.** $2\sqrt{2}-1$
c. $\sqrt{7}-1$ **d.** $1/2\sqrt{2}$

12. The negative of the statement $\sim p \wedge (p \vee q)$ is

a. $\sim p \vee q$ **b.** $p \vee \sim q$
c. $\sim p \wedge q$ **d.** $p \wedge \sim q$

13. If the curve $y = ax^2 + bx + c, x \in R$, passes through the point (1,2) and the tangent line to this curve at origin is $y = x$, then the possible values of a, b, c are

a. $a = \frac{1}{2}, b = \frac{1}{2}, c = 1$ **b.** $a = 1, b = 0, c = 1$
c. $a = 1, b = 1, c = 0$ **d.** $a = -1, b = 1, c = 1$

14. The area of the region
$R = \{(x, y): 5x^2 \le y \le 2x^2 + 9\}$ is

a. $11\sqrt{3}$ square units
b. $12\sqrt{3}$ square units
c. $9\sqrt{3}$ square units
d. $6\sqrt{3}$ square units

15. If a curve $y = f(x)$ passes through the point (1, 2) and satisfies $x\frac{dy}{dx} + y = bx^4$, then for what value of b, $\int_1^2 f(x)\, dx = \frac{62}{5}$?

a. 5 **b.** 10
c. 62.5 **d.** 31/5

16. Let $f(x)$ be a differentiable function defined on [0, 2], such that $f'(x) = f'(2-x)$, for all $x \in (0, 2)$, $f(0) = 1$ and $f(2) = e^2$. Then, the value of $\int_0^2 f(x)\, dx$ is

a. $1 - e^2$ **b.** $1 + e^2$
c. $2(1 - e^2)$ **d.** $2(1 + e^2)$

17. Let A and B be 3×3 real matrices, such that A is symmetric matrix and B is skew-symmetric matrix. Then, the system of linear equations $(A^2B^2 - B^2A^2)X = O$, where X is a 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has

a. no solution
b. exactly two solutions
c. infinitely many solutions
d. a unique solution

18. Let a, b, c be in an arithmetic progression. Let the centroid of the triangle with vertices (a, c), $(2, b)$ and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is

a. $\frac{71}{256}$ **b.** $\frac{69}{256}$ **c.** $-\frac{69}{256}$ **d.** $-\frac{71}{256}$

19. For the system of linear equations
$x - 2y = 1, x - y + kz = -2, ky + 4z = 6, k \in R$, consider the following statements

(A) The system has unique solution, if $k \ne 2, k \ne -2$.
(B) The system has unique solution, if $k = -2$.
(C) The system has unique solution, if $k = 2$.
(D) The system has no solution, if $k = 2$.
(E) The system has infinite number of solutions, if $k \ne -2$.

Which of the following statements are correct ?

a. (C) and (D) **b.** (B) and (E)
c. (A) and (E) **d.** (A) and (D)

20. The probability that two randomly selected subsets of the set {1, 2, 3, 4, 5} have exactly two elements in their intersection, is

a. $65/2^7$ **b.** $65/2^8$
c. $135/2^9$ **d.** $35/2^7$

Section B : Numerical Type Questions

21. For integers n and r, let $\binom{n}{r} = \begin{cases} {}^nC_r, & \text{if } n \ge r \ge 0 \\ 0, & \text{otherwise} \end{cases}$

The maximum value of k for which the sum, $\sum_{i=0}^{k} \binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i}\binom{13}{k+1-i}$ exists, is equal to ....

22. Let λ be an integer. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is ....

23. If $a + \alpha = 1, b + \beta = 2$ and $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}, x \ne 0$, then the value of expression $\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}}$ is ....

24. Let a point P be such that its distance from the point (5, 0) is thrice the distance of P from the point (−5, 0). If the locus of the point P is a circle of radius r, then $4r^2$ is equal to ....

25. If the area of the triangle formed by the positive X-axis, the normal and the tangent to the circle $(x-2)^2 + (y-3)^2 = 25$ at the point (5, 7) is A, then $24A$ is equal to ....

26. If the variance of 10 natural numbers 1, 1, 1,....., 1, k is less than 10, then the maximum possible value of k is ....

27. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1 and the third term is α, then 2α is ....

28. The students $S_1, S_2, \ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then, the total number of possibilities of forming such groups is ....

29. Let $i = \sqrt{-1}$. If $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}} + \frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}} = k$ and $n = [|k|]$ be the greatest integral part of $|k|$. Then, $\sum_{j=0}^{n+5}(j+5)^2 - \sum_{j=0}^{n+5}(j+5)$ is equal to ....

30. The number of the real roots of the equation $(x+1)^2 + |x-5| = \frac{27}{4}$ is ....

Answers

Physics

1. *(b)*	2. *(c)*	3. *(d)*	4. *(d)*	5. *(d)*	6. *(a)*	7. *(a)*	8. *(c)*	9. *(a)*	10. *(b)*
11. *(a)*	12. *(a)*	13. *(a)*	14. *(b)*	15. *(a)*	16. *(c)*	17. *(b)*	18. *(d)*	19. *(d)*	20. *(b)*
21. *(2)*	22. *(5)*	23. *(8)*	24. *(2)*	25. *(400)*	26. *(226)*	27. *(8)*	28. *(900)*	29. *(8)*	30. *(667)*

Chemistry

1. *(b)*	2. *(d)*	3. *(d)*	4. *(d)*	5. *(c)*	6. *(b)*	7. *(d)*	8. *(c)*	9. *(d)*	10. *(b)*
11. *(b)*	12. *(d)*	13. *(a)*	14. *(c)*	15. *(c)*	16. *(d)*	17. *(d)*	18. *(b)*	19. *(c)*	20. *(a)*
21. *(3)*	22. *(1)*	23. *(8)*	24. *(5)*	25. *(1)*	26. *(3776)*	27. *(141)*	28. *(81)*	29. *(243)*	30. *(855)*

Mathematics

1. *(b)*	2. *(a)*	3. *(c)*	4. *(d)*	5. *(d)*	6. *(b)*	7. *(d)*	8. *(b)*	9. *(d)*	10. *(b)*
11. *(a)*	12. *(b)*	13. *(c)*	14. *(b)*	15. *(b)*	16. *(b)*	17. *(c)*	18. *(d)*	19. *(d)*	20. *(c)*
21. *(*)*	22. *(1)*	23. *(2)*	24. *(56.25)*	25. *(1225)*	26. *(11)*	27. *(3)*	28. *(31650)*	29. *(310)*	30. *(2)*

Note *(*) None of the option is correct.*

Solutions

PHYSICS

1. *(b)* Since, the particle is executing SHM.

Therefore, displacement equation of wave will be

$$y = A\sin\omega t$$

$$\Rightarrow \quad y/A = \sin\omega t$$

and wave velocity equation will be

$$v_y = \frac{dy}{dt} = A\omega\cos\omega t$$

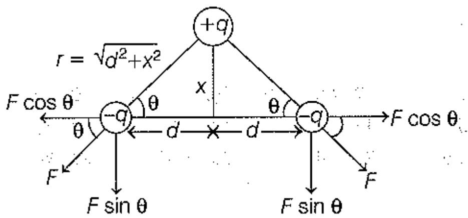
$$\Rightarrow \quad v_y/A\omega = \cos\omega t$$

Now, $\quad \sin^2\omega t + \cos^2\omega t = 1$

$$\therefore \quad (y/A)^2 + (v_y/A\omega)^2 = 1$$

This equation is similar to the equation of ellipse.

2. *(c)* The arrangement of charges is shown below



As we know that,

Coulomb's force between two charges. i.e. q_1 and q_2,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{(d^2+x^2)} \quad \text{... (i)}$$

Here, $q_1 = q_2 = q$

Force in SHM, $F = m\omega^2 x$ ...(ii)

Since, in order to have SHM $+q$ should move downwards and force responsible for this will be only

$$F' = F\sin\theta + F\sin\theta = 2F\sin\theta \quad \text{...(iii)}$$

Using Eqs. (ii) and (iii), we get

$$2F\sin\theta = m\omega^2 x$$

$$\Rightarrow \quad \frac{2}{4\pi\varepsilon_0}\frac{q^2}{(d^2+x^2)}\sin\theta = m\omega^2 x$$

$$\Rightarrow \quad \frac{2}{4\pi\varepsilon_0}\frac{q^2}{(d^2+x^2)}\cdot\frac{x}{(d^2+x^2)^{1/2}} = m\omega^2 x$$

$$\Rightarrow \quad \omega = \left(\frac{1}{2\pi\varepsilon_0}\frac{q^2}{(d^2+x^2)^{3/2}m}\right)^{1/2}$$

As, $\quad x << d$

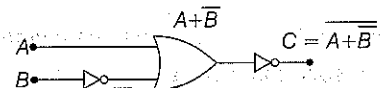
$$\therefore \quad \omega = \left(\frac{1}{2\pi\varepsilon_0}\frac{q^2}{md^3}\right)^{1/2}$$

3. *(d)* On the basis of kinetic theory of gases, the gas exerts pressure because its molecules contain uniform speed, random motion and perform elastic collision with each other, as well as with the walls of container. As a result of which gaseous molecules suffer change in momentum when impinge on the walls of container.

4. *(d)* The magnetic susceptibility (χ) of ferromagnetic material is in the range of 10^3 to 10^4, so it gets easily magnetised and demagnetised in the presence of external field region.

Therefore, when a soft ferromagnetic material is placed in an external magnetic field region, the magnetic domains may increase or decrease in size and change its orientation.

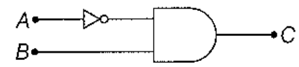
5. *(d)* The logic circuit is given as



By using De-morgan's theorem,

$$C = \overline{A+\overline{B}} = \overline{A}\cdot\overline{\overline{B}} = \overline{A}\cdot B$$

This relation can be shown by the circuit drawn below



6. *(a)* Given, $T = 2\pi\sqrt{\frac{L}{g}}$ ... (i)

where, time period, $T = 1.95$ s

Length of string, $l = 1$ m

Acceleration due to gravity $= g$

Error in time period, $\Delta T = 0.01\text{ s} = 10^{-2}$ s

Error in length, $\Delta L = 1\text{ mm} = 1\times10^{-3}$ m

Squaring Eq. (i) on both sides, we get

$$T^2 = 4\pi^2\frac{L}{g}$$

$$\Rightarrow \quad g = 4\pi^2\frac{L}{T^2}$$

$$\Rightarrow \quad \frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T} = \frac{10^{-3}}{1} + \frac{2\times10^{-2}}{1.95}$$

$$= 10^{-3} + 1.025\times10^{-2}$$

$$= 10^{-3} + 10.25\times10^{-3}$$

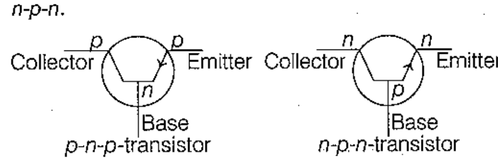
$$= 11.25\times10^{-3}$$

$$\because \quad \Delta g/g\times100 = 11.25\times10^{-3}\times10^2$$

$$= 1.125\% \simeq 1.13\%$$

7. *(a)* Transistor is a device used as switch or amplifier.

It is made by sandwiching three semiconductors, i.e. *p-n-p* and *n-p-n*.



Hence, Statement I is false, because we cannot make transistor from diode.

As we know that, amplification factor (β) is the ratio of collector current to base current.

$$\therefore \quad \beta = \frac{I_C}{I_B}$$

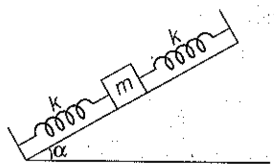
Hence, Statement II is true.

Therefore, option (a) is correct.

8. *(c)* Let T be the time period of oscillation, then

$$T = 2\pi\sqrt{\frac{M}{k_{eq}}}$$

$\therefore$ $$T = 2\pi\sqrt{\frac{M}{2k}} \quad [\because k_{eq} = k + k]$$



and frequency $(f) = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{2k}{M}}$

9. *(a)* Given, resistance, $R = 2\Omega$,

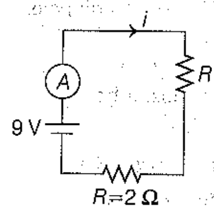
Inductance, $L = 2$ mH,

emf, $E = 9$ V

and i be the current.

$\because$ At $t = 0$ when switch is closed, inductors behave as open circuit.

$\therefore$ Effective circuit will be



By using Ohm's law, $V = i\,R_{eq}$

$\Rightarrow$ $$i = V/R_{eq}$$

where, R_{eq} is equivalent resistance of series resistors,

i.e., $$R_{eq} = R + R = 2R = 2 \times 2 = 4\Omega$$

$\therefore$ $$i = \frac{9}{4} = 2.25\,\text{A}$$

10. *(b)* Let λ_p, λ_α, m_p, m_α, v_p, v_α, p_p and p_α be the wavelength, mass, velocity and momentum of proton and α-particle, respectively.

Given, $\lambda_p = \lambda_\alpha$

As we know that,

$$\lambda = h/p$$

$\therefore$ $$\frac{h}{p_p} = \frac{h}{p_\alpha}$$

$\Rightarrow$ $$p_p = p_\alpha$$

$\Rightarrow$ $$m_p v_p = m_\alpha v_\alpha$$

$\Rightarrow$ $$m_p v_p = 4m_p v_\alpha \quad (\because m_\alpha = 4m_p)$$

$\Rightarrow$ $$\frac{v_p}{v_\alpha} = \frac{4}{1} \text{ or } 4:1$$

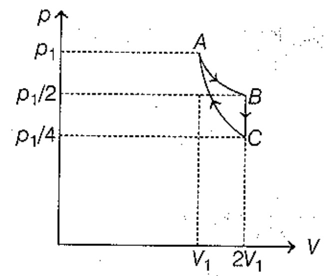
11. *(a)* Let p_i, p_f, V_i and V_f be the initial and final pressure and volume.

Given, AB is isothermal $(\Delta T = 0)$,

BC is isochoric $(\Delta V = 0)$

and CA is adiabatic $(\Delta Q = 0)$

Since, isothermal work $(W_{AB}) = p_1V_1 \ln\frac{V_f}{V_i}$



where, V_i and V_f are volume at A and B, respectively.

$\therefore$ $$W_{AB} = p_1V_1\ln\frac{2V_1}{V_1} = p_1V_1\ln 2$$

Since, at constant volume, work done is zero.

$\therefore$ $$W_{BC} = 0$$

Since, W_{CA} is an adiabatic work done, i.e.

$$W_{CA} = \frac{1}{1-\gamma}(p_fV_f - p_iV_i)$$

$\Rightarrow$ $$W_{CA} = \frac{1}{1-\gamma}\left(p_1V_1 - \frac{p_1}{4} \times 2V_1\right)$$

$$= \frac{1}{1-\gamma}(p_1V_1 - p_1V_1/2) = \frac{1}{1-\gamma}\frac{p_1V_1}{2}$$

$\therefore$ Net work done, $W_{net} = W_{AB} + W_{BC} + W_{CA}$

$$= p_1V_1\ln 2 + 0 + \frac{1}{1-\gamma}\frac{p_1V_1}{2}$$

$$= p_1V_1[\ln 2 + 1/2(1-\gamma)]$$

From ideal gas law, $pV = nRT$

$\therefore$ $W_{net} = RT[\ln 2 - 1/2(\gamma - 1)]$ $\quad (\because n = 1)$

12. *(a)* Given, $V = 1.24$ million volt $= 1.24 \times 10^6$ volt

Since, energy $(E) = eV$

where, e is the charge of electron $= 1.6 \times 10^{-19}$ C

$\therefore$ $$E = 1.6 \times 10^{-19} \times 1.24 \times 10^6 \quad \text{... (i)}$$

As we know that,

Energy of photon, $$E = \frac{hc}{\lambda} \quad \text{...(ii)}$$

Here, Planck's constant, $h = 6.67 \times 10^{-34}$ J-s,

c = speed of light in free space, $c = 3 \times 10^8$ ms^{-1}

Equating Eqs. (i) and (ii), we get

$$1.6 \times 10^{-19} \times 1.24 \times 10^6 = \frac{6.67 \times 10^{-34} \times 3 \times 10^8}{\lambda}$$

$\Rightarrow$ $$\lambda = \frac{20.01 \times 10^{-13}}{1.6 \times 1.24} = 10.09 \times 10^{-13}$$

$$= 1.009 \times 10^{-12} \simeq 10^{-3} \times 10^{-9}$$

$$= 10^{-3}\,\text{nm}$$

13. *(a)* The equation of a travelling wave in standard form,

$$y = A\sin(\omega t \pm kx)$$

Only option (a), i.e. $y = A\sin(15x - 2t)$ satisfies this equation.

14. *(b)* Let n_f, n_i be the final and initial orbit.

As we know that,

$$\frac{1}{\lambda} = 1.09 \times 10^7\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

Now, checking for each option, we get

(a) $\frac{1}{\lambda} \propto \left[\frac{1}{3^2} - \frac{1}{4^2}\right] = \left[\frac{1}{9} - \frac{1}{16}\right] = 0.05$ ... (i)

(b) $\frac{1}{\lambda} \propto \left[\frac{1}{1} - \frac{1}{4}\right] = 0.75$ ... (ii)

(c) $\frac{1}{\lambda} \propto \left[\frac{1}{16} - \frac{1}{25}\right] = 0.0225$ ... (iii)

(d) $\frac{1}{\lambda} \propto \left[\frac{1}{4} - \frac{1}{9}\right] = 0.14$ ... (iv)

The option (b) has highest value.

Since, frequency, $f = \frac{c}{\lambda} \Rightarrow f \propto \frac{1}{\lambda}$

$\therefore$ Frequency will be maximum for transition $n = 2$ to $n = 1$.

15. *(a)* According to Young's double slit experiment,
The distance of nth bright fringe from the centre,

$$y_n = \frac{n\lambda D}{d}$$

Since, $\lambda_{\text{violet}} < \lambda_{\text{red}}$

$\therefore$ $y_{\text{violet}} < y_{\text{red}}$

$\therefore$ Consecutive fringe lines will come closer.

16. *(c)* Given, radius of hole, $r = a/2$
and radius of disc, $R = a$
Let x_{CM} be the centre of mass of system,
m_1, x_1 be the mass and centre of mass of disc
m_2, x_2 be the mass and centre of mass of circular hole.

$\therefore$ $m_2 = \frac{m_1}{\pi R^2}\pi r^2$

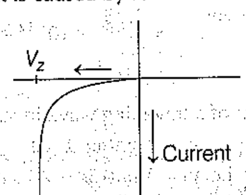
$\Rightarrow$ $m_2 = \frac{m_1 r^2}{R^2} = \frac{m_1 (a/2)^2}{a^2} = \frac{m_1}{4}$

and $x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

$\therefore$ $x_{CM} = \frac{m_1 a - (m_1/4)(3a/2)}{m_1 - (m_1/4)}$

$\Rightarrow$ $x_{CM} = \frac{m_1 a(1 - 3/8)}{3m_1/4} \Rightarrow x_{CM} = \frac{5a}{6}$

17. *(b)* As we know that, Zener breakdown takes place, when we supply reverse bias voltage to Zener diode. Due to heavily dopping, the electrons in the valence band of p-type region can jump easily to the conduction band of n-type region, hence due to high electric field, zener breakdown occur. Thus, there is very high sudden increase in Zener current (I_Z) that is caused by reverse breakdown voltage (V_Z).



Hence, Zener breakdown is easily observed in Zener diode which is heavily doped and having narrow depletion region.

18. *(d)* As we know that;

A. Source of microwave frequency is magnetron.

B. Source of infrared frequency is vibration of atoms and molecules.

C. Source of gamma ray is radioactive decay of nucleus.

D. Source of X-ray is transition of electron in inner shells.

$\therefore$ The correct match is A $\rightarrow$ (2), B $\rightarrow$ (4), C $\rightarrow$ (1), D $\rightarrow$ (3).

19. *(d)* Given, speed of projection $= v_0$

Damping force, $F = ma = -\alpha x^2$

$\Rightarrow$ $a = -\alpha x^2/m$

Also, $a = v\frac{dv}{dx}$

$\Rightarrow$ $v dv = a\,dx = -\frac{\alpha}{m}x^2 dx$

Integrating both sides, we get

$$\int_{v_0}^{v} v\,dv = \int_0^x -\frac{\alpha}{m}x^2 dx$$

$\Rightarrow$ $\left(\frac{v^2}{2}\right)_{v_0}^{0} = -\frac{\alpha}{m}\left(\frac{x^3}{3}\right)_0^x$

$\Rightarrow$ $0 - v_0^2/2 = -\frac{\alpha}{m}\frac{x^3}{3} \Rightarrow x = \left(\frac{3m}{2}\frac{v_0^2}{\alpha}\right)^{1/3}$

20. *(b)* Given, weight of body at North pole,
$w_p = mg = 49$ N
Radius of Earth, $R = 6400$ km
Let weight of body at equator be w_e.
At equator, $g_e = g - R\omega^2$

$\therefore$ $w_e = mg_e = m(g - R\omega^2)$

Since, $w_p > w_e \Rightarrow w_e < 49$ N
Hence, above condition is satisfied by only option (b).

21. *(2)* Let initial length and diameter be l_1 and d_1, whereas final length and diameter be l_2 and d_2.
Given, $l_2 = 2l_1, d_2 = 2d_1, \Delta l_1 = 0.04$ m
By using formula of Young's modulus of elasticity,

$$Y = \frac{F \cdot l}{A\Delta l}$$

$\because$ $Y_1 = Y_2$

$\Rightarrow$ $\frac{Fl_1}{A_1 \times \Delta l_1} = \frac{Fl_2}{A_2 \times \Delta l_2}$

$\Rightarrow$ $\frac{Fl_1}{\pi(d_1/2)^2 \times 0.04} = \frac{F2l_1}{\pi(2d_1/2)^2 \Delta l_2}$

$\Rightarrow$ $\frac{1}{1/4 \times 0.04} = \frac{2}{\Delta l_2}$

$\Rightarrow$ $\Delta l_2 = 0.02$ m $= 2$ cm

22. *(5)* Given, radius of cylindrical wire, $r = 0.5$ mm $= 0.5 \times 10^{-3}$ m
Conductivity, $\sigma = 5 \times 10^7$ S/m
Electric field, $E = 10$ mV/m $= 10 \times 10^{-3}$ V/m
We know that current density,

$\therefore$ $J = \sigma E$
$= 5 \times 10^7 \times 10 \times 10^{-3} = 5 \times 10^5 \text{ A/m}^2$

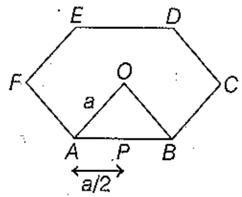
Also, $J = I/A \Rightarrow I = JA$

$\Rightarrow$ $I = 5 \times 10^5 \times \pi \times (0.5 \times 10^{-3})^2$
$= 5 \times 10^5 \times \pi \times 25 \times 10^{-8} = 125\,\pi \times 10^{-3}$

$\Rightarrow$ $x^3\,\pi\text{mA} = 125\,\pi\text{ mA} \Rightarrow x^3 = 5^3$

$\Rightarrow$ $x = 5$

23. *(8)* Given, mass of uniform bar, $m = 6$ kg
Length of bar, $l = 2.4$ m



Side of hexagon, $a = \frac{2.4}{6} = 0.4$ m

Mass of each side, $m' = 6/6 = 1$ kg

and $OP = \sqrt{a^2 - a^2/4} = a\sqrt{3}/2$

$= 0.2\sqrt{3}$m

Now, by using parallel axes theorem,

$$I_{OP} = \frac{m'a^2}{12} + m'\left(\frac{\sqrt{3}a}{2}\right)^2$$

$$= \frac{a^2}{12} + \frac{3a^2}{4} = \frac{10a^2}{12} = \frac{5a^2}{6}$$

$\Rightarrow$ $I_{OP} = \frac{5}{6} \times 0.4 \times 0.4$

and I_{net} (net moment of inertia at O)

$$= 6 \times \frac{5}{6} \times 0.4 \times 0.4 = 8 \times 10^{-1} \text{ kg-m}^2$$

24. *(2)* Given, $m_A = 1$ kg, $m_B = 2$ kg, $(KE)_A : (KE)_B = A : 1$

Linear momentum of A and B are equal.

$\Rightarrow$ $p_A = p_B$

$\because$ Kinetic energy (KE) $= p^2/2m$

$\therefore$ $\frac{KE_A}{KE_B} = \frac{m_B}{m_A} = \frac{2}{1} = \frac{A}{1} \Rightarrow A = 2$

25. *(400)* Given, $T_1 = 27°C = 27 + 273 = 300$ K,

$p_1 = 1$ atm, $v_1 = 200$ ms^{-1}, $T_2 = 127°C = 400$ K,

$p_2 = 2$ atm, $v_2 = ?$

As we know that,

Root mean square speed, $v_{rms} = \sqrt{\frac{3RT}{m}}$

$\therefore$ $\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{300}{400}} = \sqrt{\frac{3}{4}}$

$\Rightarrow$ $v_2 = \sqrt{\frac{4}{3}} v_1 = \frac{2}{\sqrt{3}} \times 200 = \frac{400}{\sqrt{3}}$ ms^{-1}

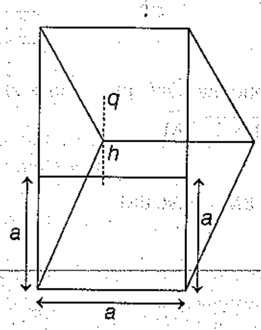
$\Rightarrow$ $\frac{x}{\sqrt{3}} = \frac{400}{\sqrt{3}} \Rightarrow x = 400$

26. *(226)* Given, charge, $q = 12\ \mu C = 12 \times 10^{-6}$C

Height of charge from surface, $h = 6$ cm $= 6 \times 10^{-2}$ m

and side of square, $a = 12$ cm $= 12 \times 10^{-2}$ m

From figure, it is clear that the given square is one of the face of a cube of side 12 cm and + 12μC charge is placed at its centre. Then, by Gauss's theorem,



Flux through any face, $\phi = \frac{q}{6\varepsilon_0}$

$\Rightarrow$ $\phi = \frac{12 \times 10^{-6}}{6 \times 8.854 \times 10^{-12}} = 0.226 \times 10^6$ N-m^2/C

$= 226 \times 10^3$ N-m^2/C

27. *(8)* Given, power of transmitted signal, $P_i = 0.1$ kW $= 0.1 \times 10^3$ W $= 10^2$ W

Rate of attenuation, $R = -5$ dB/km

Length of cable, $l = 20$ km

Power received at receiver, $P_x = 10^{-x}$ W

Total loss, $\beta = R \times l = -5 \times 20 = -100$ dB

$\because$ Gain (β) $= 10 \log_{10} \frac{P_0}{P_i}$

$\therefore$ $\beta = -100 = -10 \log_{10} \frac{P_0}{P_i}$

$\Rightarrow$ $-10 = \log_{10} \frac{P_0}{P_i} \Rightarrow 10^{-10} = \frac{P_0}{P_i}$

$\Rightarrow$ $P_0 = 10^{-10} P_i = 10^{-10} \times 10^2 = 10^{-8} \Rightarrow P_0 = 10^{-8}$ W

Hence, $x = 8$

28. *(900)* Given, angular frequency at resonance, $\omega_0 = 10^5$rads^{-1}

Power drawn from circuit, $P = 16$ W

and supply voltage, $V = 120$ V

Let resistance of circuit $= R$.

As, $P = V^2/R$

$\Rightarrow$ $R = V^2/P = \frac{120 \times 120}{16}$

$= 30 \times 30 = 900\ \Omega$

29. *(8)* Given, $v_A = v_B = 7.2$ kmh^{-1}

$= \frac{72}{10} \times \frac{5}{18} = 2$ ms^{-1}

Frequency of source, $f_s = 676$ Hz

Speed of sound in air, $v = 340$ ms^{-1}

Let f_0 be the frequency heard by each driver.

By using Doppler effect for A,

$(v - v_A) f_s = (v + v_B) f_0$

$\Rightarrow$ $f_0 = \left(\frac{v + v_A}{v - v_B}\right) f_s = \left(\frac{340 + 2}{340 - 2}\right) 676 = \frac{342}{338} \times 676 = 684$ Hz

Now, beat frequency $= f_0 - f_s = 684 - 676 = 8$ Hz

30. *(667)* Given, frequency of wave, $f = 3$ GHz $= 3 \times 10^9$ Hz

Relative permittivity, $\varepsilon_r = 2.25$

Since, $f = c/\lambda$

$\Rightarrow$ $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = 0.1$ m

$\because \lambda_m$ (wavelength of wave in a medium) $= \lambda/\mu$

and as we know that, $\mu = \sqrt{\mu_r \varepsilon_r}$

As, dielectric is non-magnetic, $\mu_r = 1$

$\Rightarrow$ $\mu = \sqrt{2.25} = 1.5$

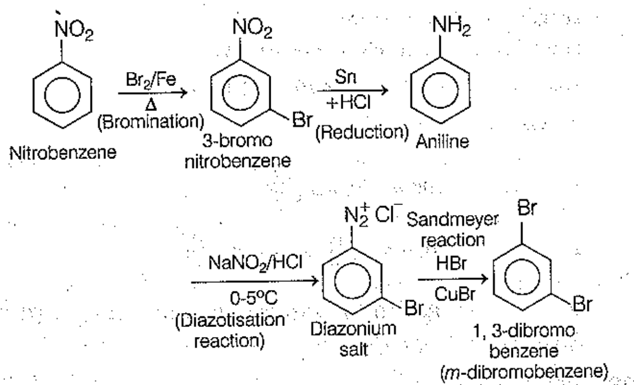
$\Rightarrow$ $\lambda_m = \frac{0.1}{1.5} = \frac{1}{15} = 0.0667$ m

$= 6.67$ cm $= 667 \times 10^{-2}$ cm

CHEMISTRY

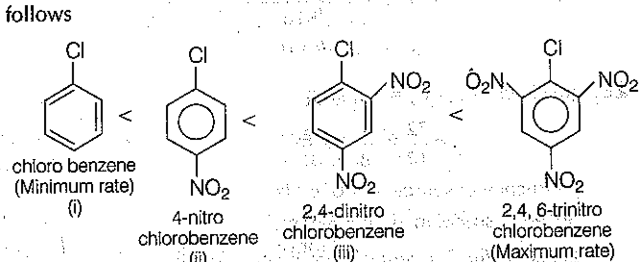
1. (*b*) In first step, NO_2 group is electron withdrawing group, it decreases the electron density in *meta* position and bromination occur in *meta* position. In second step, Sn/HCl used for reduction to convert $—NO_2$ to $—NH_2$ and form aniline. In next step, diazonium salt is formed and at last bromine replace $N_2^+Cl^-$ group to form 1, 3-dibromobenzene.

   Complete reaction is as follows



2. (*d*) According to Hardy-Schulze rule, for negatively charged sol, most (+ve) charged ion is needed for efficient coagulation. Blood is a negatively charged sol. Hence $FeCl_3$ can be used for blood clotting and it from Fe^{3+} ion.

3. (*d*) Rate of nucleophilic substitution in aromatic halide are as follows



   Greater the number of nitro $(—NO_2)$ group in chlorobenzene, greater the rate of nucleophilic substitution reaction in aromatic halides.

   When a nitro group is present at *ortho* and *para* position, it can withdraw electrons from the benzene ring. This facilitates, the attack of nucleophiles on haloarenes.

4. (*d*) According to Bohr's theory,

   I. $KE \propto \frac{Z^2}{n^2}$ or $13.6 \propto \frac{Z^2}{n^2}\frac{(eV)}{(atom)}$, (∴ Correct)

   II. Speed of electron $\propto \frac{Z}{n}$

   (Here, Z = atomic number, n = number of shells)

   ∴ $v \times n \propto Z$ (∴ Incorrect)

   III. Frequency of revolution of electron $= \frac{v}{2\pi r}$

   $\text{Frequency} \propto \frac{Z^2}{n^3}$ $\left(\because v \propto \frac{z}{n}, r \propto \frac{n^2}{z}\right)$ (∴ Incorrect)

   IV. $F = \frac{kq_1q_2}{r^2} = \frac{kZe^2}{r^2}$

   $$F = \frac{Z}{\left(\frac{n^2}{Z}\right)^2}$$

   $$F \propto \frac{Z^3}{n^4}$$ (∴ Correct)

   Hence, only I, and IV are correct.

5. (*c*) (A) Alkyl chloride reacts with H_2/Pd-$BaSO_4$ and reduced to alkyl aldehyde. This is known as Rosenmund reduction.

   $$\underset{\text{Alkyl chloride}}{R—\overset{\overset{O}{\|}}{C}—Cl} \xrightarrow[\text{Pd-BaSO}_4]{H_2} \underset{\text{Alkylal dehyde}}{R—\overset{\overset{O}{\|}}{C}—H}$$

   (Rosenmund reduction)

   (B) Carboxylic acid reacts with Cl_2/P in aqueous medium to form 2-chlorocarboxylic acid. This reaction is known as HVZ reaction.

   $$\underset{\text{Alkyl carboxylic acid}}{R—CH_2—COOH} \xrightarrow[H_2O]{Cl_2/P} \underset{\text{2-chlorocarboxylic acid}}{R—\underset{Cl}{\underset{|}{CH}}—COOH}$$

   [Hell-Volhard-Zelinsky (HVZ) reaction]

   (C) Alkyl amide reacts with Br_2 in presence of NaOH to give alkyl amine. This reaction is known as Hofmann bromide reaction,

   $$\underset{\text{Alkyl amide}}{R—\overset{\overset{O}{\|}}{C}—NH_2} \xrightarrow[NaOH]{Br_2} \underset{\text{Alkyl amine}}{R—NH_2}$$

   (Hofmann bromide reaction)

   (D) Ketone reacts with Zn-Hg in presence of conc.HCl to give alkane. This reaction is known as clemmensen reduction.

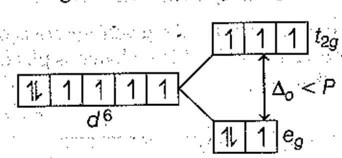
   $$\underset{\text{Ketone}}{R—\overset{\overset{O}{\|}}{C}—CH_3} \xrightarrow[\text{Conc. HCl}]{Zn(Hg)} \underset{\text{Alkane}}{R—CH_2—CH_3}$$

   (Clemmenson reduction)

   Hence, correct match is (A)–2, (B)–4, (C)–1, (D)–3.

6. (*b*) (i) $[FeCl_4]^{2-} \longrightarrow Fe^{2+} \longrightarrow [Ar]\,3d^6$

   ∴ Cl is weak field ligand so does not pairing occur



   So, magnetic moment $(\mu) = \sqrt{n(n+2)}$ BM

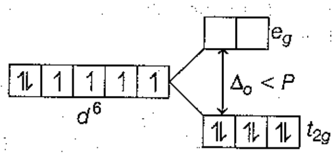
   $= \sqrt{4(4+2)}$ BM

   (n = Number of total unpaired e^- = 4)

   $= \sqrt{24}$ BM = 4.90 BM

(ii) $[Co(C_2O_4)_3]^{3-} \longrightarrow Co^{3+} \longrightarrow [Ar]\,3d^6$

C_2O_4 is strong field ligand so pairing occur.



All electrons are paired, $n = 0$

hence, $\mu = 0$

(iii) $MnO_4^{2-} \rightarrow Mn^{+6} \rightarrow [Ar]3d^1$

$n = 1$

$= \sqrt{n(n+2)}$ BM

$= \sqrt{1(1+2)}$ BM

$= \sqrt{3}$ BM

$= 1.73$ BM

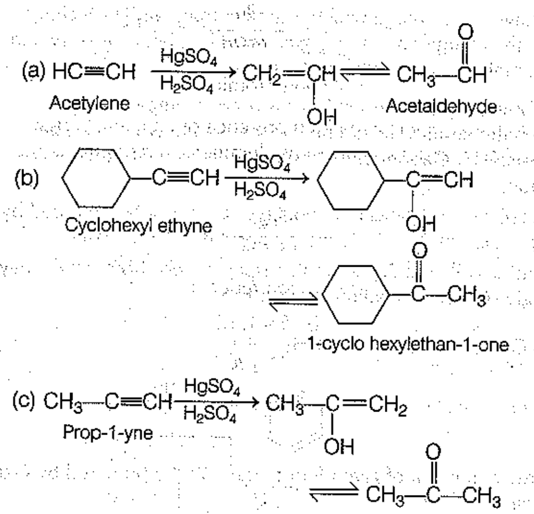
7. (*d*)

Alkali metal	**Colour (Flame)**	**λ (nm)**
Li	Crimson red	670.8
Na	Yellow	589.2
Rb	Red, violet	780.0
Cs	Blue	455.5

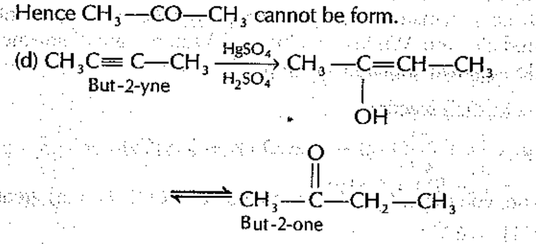
Alkali metals have very low value of ionisation energy as compared to other metals. So, alkali metals easily get excited and impart colour to flame.

Hence, Rb is most excited and having high value of wavelength in all alkali metals.

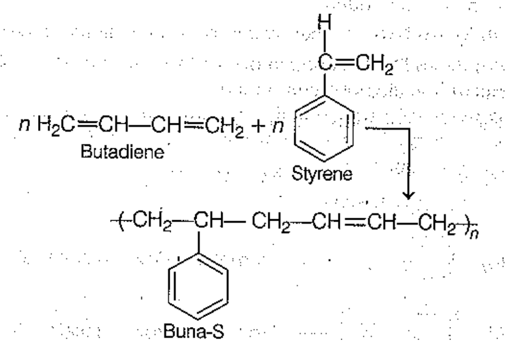
8. (*c*) Reaction of $HgSO_4$/dil. H_2SO_4 with alkyne result in addition of water as per Markownikoff's rule.



Hence $CH_3—CO—CH_3$ cannot be form.



9. (*d*) Buna-S is the co-polymer of but 1, 3-diene and styrene as follows.



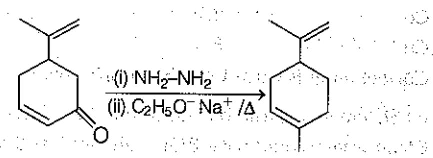
Styrene butadiene rubber is also called buna-S, in which 'Bu' stands for butadiene, 'na' stands for sodium and 'S' stands for styrene.

Since, butadiene and styrene is one of the constituent monomer of given polymer.

10. (*b*) To reduce the carbonyl groups into alkane, wolff-Kishner reduction is used, without affecting the double bond.

Wolff-Kishner reagent It utilises hydrazine ($NH_2—NH_2$) as the reducing agent in the presence of strong base KOH or $C_2H_5\bar{O}Na^+$ in a high boiling protic solvent.

The driving force for the reaction is the conversion of hydrazine to nitrogen gas.



11. (*b*) (A) Valium – (4) Tranquilizer

A tranquilizer drug became a standard drug for the treatment of anxiety and one of most commonly prescribed drugs of all time.

(B) Morphine – (3) Analgesic

Morphine is effective for both acute and chronic pain and often used before and after surgery.

(C) Norethindrone – (1) Antifertility drug

It is a form of progesterone, a female hormone important for regulating ovulation and menstruation.

(D) Vitamin B_{12}–(2) Pernicious anaemia

It is a nutrient that helps to keep our body blood cells and nerve cells healthy and help in making DNA.

12. (*d*) (A) Siderite $\rightarrow FeCO_3$

Siderite is a mineral composed of iron (II) carbonate ($FeCO_3$).

(B) Calamine $\rightarrow ZnCO_3$

Calamine is used to treat mild itchiness and contain zinc in its formula.

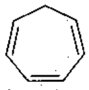
(C) Kaolinite $\rightarrow Al_2(OH)_4 \cdot Si_2O_5$

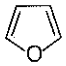
Kaolinite is a type of clay found in nature. It is a dioctahedral pyrosilicate clay containing Si_2O_5.

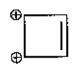
(D) Malachite $\rightarrow Cu(OH)_2 \cdot CuCO_3$

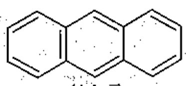
Malachite is a copper carbonate hydroxide mineral, with the formula $Cu_2CO_3(OH)_2$.

13. (*a*) Compound, which obey Huckel rule, $(4n+2)\pi$ is called aromatic compound.
Compound, which obey $4n\pi$ rule, is called anti-aromatic compound.
Compound which contain one or more sp^3 carbon in its structure is called non-aromatic compound.
The nature of following ions/compounds are as follows :

(a)  sp^3 Carbon $\longrightarrow$ Non-aromatic

(b)  ($6e^-$) $\longrightarrow$ Aromatic (follow Huckel's rule)

(c)  ($2e^-$) $\longrightarrow$ Aromatic (follow Huckel's rule)

(d)  $\longrightarrow$Anthracene$\longrightarrow$ Aromatic
($14e^-$)

Only (a) is non-aromatic and rest all are aromatic.

14. (*c*) Generally, due to decrease in metallic radius and increase in atomic mass density increase across the period from left to right.

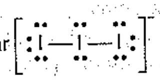
Metal	Density (g/cm^3)
Zn	7.13
Cr	7.19
Fe	7.8
Co	8.7
Cu	8.9

Correct order is Cu > Co > Fe > Cr > Zn.

15. (*c*) Statement I is true but statement II is false.
Clean water would have BOD value of less than 5 ppm whereas highly polluted water could have a BOD value of 17 ppm or more.
Hence, the value of parameter 'BOD' is important for survival of aquatic life but optimum value of BOD is 17 ppm or more. So, statement II is incorrect.

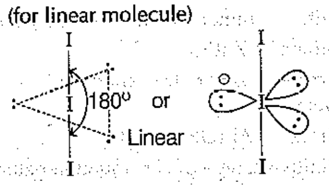
16. (*d*) Red colour of ruby is due to presence of Cr^{3+} ions in Al_2O_3.
Chromium is the trace element that causes ruby's red colour, which ranges from an orange red to a publish red. The strength of ruby's red depends on how much chromium is present.

17. (*d*) Hybridisation of central I in I_3^- is sp^3d with 3 lone pair and 2 bond pair.

Shape Linear 

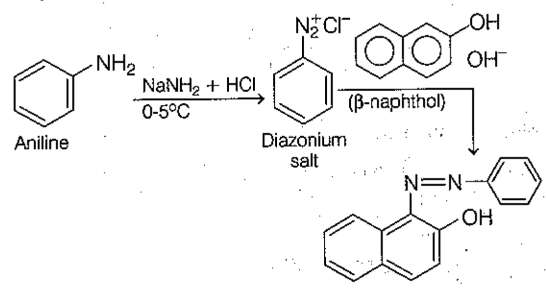
Lone pair 3 lone pair

Bond angle 180° (for linear molecule)



18. (*b*) Both A and R are true and R is the correct explanation of A.
As we know, hydrogen is most abundant element in the universe but the most abundant gas in the troposphere is nitrogen, because hydrogen is lightest element.
Troposphere contains 3-quarters-of-mass-of-the-entire-atmosphere. The air here contain 78% nitrogen and 21% oxygen. The last 1% is made of argon, water vapour and carbon dioxide.

19. (*c*) Initially aniline reacts with diazotisation reagent to form diazonium salt. Then β-naphthol react with salt and orange-red dye is obtained. So, diazonium salt of aniline is used to prepare orange-red dye.



20. (*a*) Correct option is (a) i.e. LiF > LiCl; MgO > NaCl. Melting point is directly proportional to lattice energy. Lattice energy is the energy required to separate a mole of an ionic solid into gaseous ions.
It depends upon charge of ions and size of ions.

$$M.P \propto L.E \propto \frac{\text{Charge}}{\text{Size}}$$

Li → +1	Li → +1
F = −1	Cl → −1

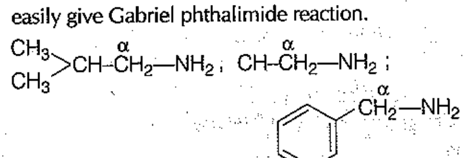
Both LiF and LiCl having same charge, so melting point will depend on size.
Larger the size of anion, lesser the lattice energy and hence, melting point order is LiF > LiCl.

Similarly,

MgO	NaCl
Mg → 2 +	Na → 1+
O → 2 −	Cl → 1−

MgO having +2 charge which is greater than NaCl (+1) charge. So, greater the charge on the ions greater will be lattice energy and hence, melting point order is MgO > NaCl.

21. (*3*) Gabriel phthalimide synthesis is used to prepare 1° aliphatic or alicyclic amine. Hence, amine which can be synthesised by Gabriel phthalimide synthesis method contains α-carbon.
Aniline ($C_6H_5NH_2$) does not contain α-C cannot be prepared by Gabriel reaction.
Rest amines all contain α-C in its respective position, hence they can easily give Gabriel phthalimide reaction.



∴ Three amins out of given four anions can be prepared by Gabriel synthesis.

22. (*1*) Only S_2-form of sulphur is paramagnetic in nature. Because S_2 is like O_2 i.e. paramagnetic as per molecular orbital theory. It contains unpaired electron. While α-sulphur and β-sulphur are diamagnetic as they do not have unpaired electron.

23. (*8*) Combustion reaction :

$$C_xH_y(g) + \left(x + \frac{y}{4}\right)O_2(g) \longrightarrow xCO_2(g) + \frac{y}{z}H_2O(l)$$

Suppose, volume of C_xH_y is V and volume of O_2 is 6 times greater than C_xH_y = 6 V
then volume of $xCO_2 \Rightarrow Vx = 4V$
$x = 4$

Since, $V_{O_2} = 6 \times V_{C_xH_y}$

$$V\left(x + \frac{y}{4}\right) = 6\text{ V}$$

$$\left(x + \frac{y}{4}\right) = 6 \quad \text{... (i)}$$

Put value of $x = 4$ in Eq. (i)

We get, $4 + \frac{y}{4} = 6 \Rightarrow y = 8$

24. (*5*) Given, mass of $C_2H_2(g) = 4.75$ g

Molecular weight = 26 g/mol

Temperature = 50 + 273 = 323 K

Pressure = 740 torr/mm of Hg

$$\text{Pressure} = \frac{740}{760}\text{ atm}$$

$$R = 0.0821\text{ L atm mol}^{-1}\text{ K}^{-1}$$

Hence, no. of mole $n = \frac{4.75}{26}$ mol

Formula used, $pV = nRT$ (ideal gas)

$$\Rightarrow V = \frac{nRT}{p} = \frac{4.75}{26} \times \frac{0.0821 \times 323}{(740/760)}$$

$$= \frac{96314.078}{19240} = 5.0059\text{ L} = 5\text{ L}$$

25. (*1*) Pure solvent : $C_6H_6(l)$

Given, $T_f^\circ = 5.5°C$

$K_f = 5.12°$ C/m $\Rightarrow m = 200$ g

$m_{solute} = 10$ g

Molar mass of solute $C_4H_{10} = 12 \times 4 + 10 = 58$

Solute (C_4H_{10}) is non-dissociative;

$\therefore$ $i = 1$

$\therefore$ $\Delta T_f = iK_f m$

$$\Rightarrow (T_f^\circ - T_f^1) = 1 \times 5.12 \times \frac{(10/58)}{(200/1000)}$$

$$5.5 - T_f^1 = \frac{5.12 \times 5 \times 10}{58} \Rightarrow T_f^1 = 1.086°C$$

or $T_f^1 \approx 1°C$

26. (*3776*) Reaction, $MnO_4^- + H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O$

$n = 5$

Applying Nernst equation, $E_{cell} = E_{cell}^\circ - \frac{0.0591}{n}\log\frac{[P]}{[R]}$

or $$E_{cell} = E_{cell}^\circ - \frac{0.0591}{5}\log\frac{[Mn^{2+}]}{[MnO_4^-]}\left[\frac{1}{H^+}\right]^8$$

(I) Given, $[H^+] = 1$ M

$$E_1 = E^\circ - \frac{0.0591}{5}\log\frac{[Mn^{2+}]}{[MnO_4^-]}$$

(II) Now, $[H^+] = 10^{-4}$ M

$$E_2 = E^\circ - \frac{0.0591}{5}\log\frac{[Mn^{2+}]}{[MnO_4^-]} \times \frac{1}{(10^{-4})^8}$$

$\therefore$ $|E_1 - E_2|$

$$|E_1 - E_2| = \frac{0.0591}{5} \times 32 = 0.3776\text{ V} = 3776 \times 10^{-4}$$

$x = 3776$

27. (*141*) Given, $[K_{sp}]_{PbI_2} = 8 \times 10^{-9}$

To calculate solubility of PbI_2 in 0.1 M solution of $Pb(NO_3)_2$.

(I) $Pb(NO_3)_2 \longrightarrow Pb^{2+}(aq) + 2NO_3^-(aq)$

0.1 M 0.1 M 0.2 M

(II) $PbI_2(s) \rightleftharpoons Pb^{2+}(aq) + 2I^-(aq)$

S 2S

$\therefore$ $[Pb^{2+}] = S + 0.1 \approx 0.1$

$\because$ $S << 0.1$

Now, $K_{sp} = 8 \times 10^{-9}$

$$[Pb^{2}][I^-]^2 = 8 \times 10^{-9}$$

$$0.1 \times (2S)^2 = 8 \times 10^{-9}$$

$$4S^2 = 8 \times 10^{-8} \Rightarrow S = 141 \times 10^{-6}\text{ M}$$

$x = 141$

28. (*81*) Given,

$$\underset{\text{Sucrose}}{C_{12}H_{22}O_{11}} + H_2O \xrightarrow{\text{1st order}} \underset{\text{Glucose}}{C_6H_{12}O_6} + \underset{\text{Fructose}}{C_6H_{12}O_6}$$

$$t_{1/2} = \frac{10}{3}\text{ h}$$

$t = 0$, $a = [A]_0$ (initial conc.)

At $t = 9$ h $a - x = [A]_t$ [conc. at time t]

For using 1st order equation,

$$K = \frac{2.303}{t}\log\frac{[A]_0}{[A]_t} \Rightarrow \frac{K \times t}{2.303} = \log\frac{[A]_0}{[A]_t}$$

$$\frac{\ln 2 \times 9}{10/3 \times 2.303} = \log\left(\frac{1}{F}\right) \Rightarrow \log\left(\frac{1}{F}\right) = 0.8124 \quad \left(\because k = \frac{\ln 2}{t_{1/2}}\right)$$

$$\log\left(\frac{1}{F}\right) = 81.24 \times 10^{-2}$$

$x = 81.24$ or $x \approx 81$

29. (*243*) Reaction

$$\underset{\substack{\text{Aniline}\\ \text{Mol. wt} = 98}}{C_6H_5NH_2} \xrightarrow[\text{Efficiency}]{90\%} \underset{\substack{\text{Acetanilide}\\ \text{(mol. wt.} = 135)}}{C_6H_5-\underset{H}{\underset{|}{N}}-\overset{O}{\overset{\|}{C}}-CH_3}$$

Given, weight = 1.86 g

Here, 1 mole of aniline gives 1 mole of acetanilide

$\therefore$ mole of aniline = mole of acetanilide

$$\Rightarrow \frac{1.86}{93} = \frac{W_{Acetanilide}}{135}$$

$$W_{Acetanilide} = \frac{1.86 \times 135}{93}\text{ g} = 2.70\text{ g}$$

But efficiency of reaction is 90% only.

Hence, mass of acetanilide produced

$$= 2.70 \times \frac{90}{100}\text{ g} = 2.43\text{ g} = 243 \times 10^2\text{ g}$$

$x = 243$

30. (*855*) Reaction, $\underset{\text{Acetylene}}{3HC\equiv CH(g)} \longrightarrow \underset{\text{Benzene}}{C_6H_6(l)}$

Given, $\Delta G_f^\circ = (CH \equiv CH) = -2.04 \times 10^5$ J mol^{-1}

$\Delta G_f^\circ(C_6H_6) = -1.24 \times 10^5$ J mol^{-1}

Gibb's free energy, $\Delta G^\circ = -nRT\ln K$

$$\Delta G^\circ = \Sigma(\Delta G_f^\circ)_P - \Sigma(\Delta G_f^\circ)_R$$

$$-nRT\ln K = -n'RT\ln K_p - (-n''RT\ln K_r)$$

$$\Rightarrow -RT\ln K = 1 \times (-1.24 \times 10^5) - (-3 \times 2.04 \times 10^5)$$

$$-2.303 \times R \times T\log K = 4.88 \times 10^5$$

$$\log K = -\frac{4.88 \times 10^5}{2.303 \times 8.314 \times 273}$$

$n\ln K = n'\ln K_p - (-n''\ln K_r)$

$K = 85.52 \Rightarrow K = 855 \times 10^{-1}$

$x = 855$

MATHEMATICS

1. *(b)* Given statements,

(A) $[\sim q \wedge (p \rightarrow q)] \rightarrow \sim p$

(B) $[(p \vee q) \wedge \sim p] \rightarrow q$

For statement (A),

$p \rightarrow q$	$\sim q \wedge (p \rightarrow q)$	$[\sim q \wedge (p \rightarrow q)] \rightarrow \sim p$
T	F	T
F	F	T
T	F	T
T	T	T

p	q	$\sim p$	$\sim q$
T	T	F	F
F	T	T	F

p	q	$\sim p$	$\sim q$
T	F	F	T
F	F	T	T

$\therefore$ Statement (A) is tautology.

For statement (B),

$p \vee q$	$(p \vee q) \wedge \sim p$	$[(p \vee q) \wedge \sim p] \rightarrow q$
T	F	T
T	F	T
T	T	T
F	F	T

$\therefore$ Statement (B) is tautology.

$\therefore$ (A) and (B) both are tautologies.

2. *(a)* Given, $P(a, 6, 9)$

Equation of line $\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$

Image of point P with respect to line is point $Q(20, b, -a-9)$

Mid-point of P and $Q = \left(\frac{a+20}{2}, \frac{6+b}{2}, \frac{-a}{2}\right)$

This point lies on line

$$\therefore \quad \frac{\frac{a+20}{2}-3}{7} = \frac{\frac{6+b}{2}-2}{5} = \frac{\frac{-a}{2}-1}{-9}$$

$$\Rightarrow \quad \frac{a+14}{14} = \frac{b+2}{10} = \frac{a+2}{18}$$

$$\Rightarrow \quad \frac{a+14}{14} = \frac{a+2}{18} \text{ and } \frac{b+2}{10} = \frac{a+2}{18}$$

Solving, we get $a = -56, b = -32$

$\therefore \quad |a+b| = |-56-32| = 88$

3. *(c)* Given, point $(1, 0, 2)$

Equation of plane =

$$\mathbf{r} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 1 \text{ and } \mathbf{r} \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) = -2$$

Equation of plane passing through the intersection of given planes is

$$[\mathbf{r} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - 1] + \lambda[\mathbf{r} \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + 2] = 0$$

$\because$ This plane passes through point $(1, 0, 2)$ i.e.,

vector $(\hat{\mathbf{i}} + 2\hat{\mathbf{k}})$

$$\therefore \quad [(\hat{\mathbf{i}} + 2\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - 1] + \lambda[(\hat{\mathbf{i}} + 2\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + 2] = 0$$

$$\Rightarrow \quad (3-1) + \lambda(1+2) = 0$$

$$\Rightarrow \quad 2 + \lambda \times 3 = 0$$

$$\Rightarrow \quad \lambda = -2/3$$

Hence, equation of required plane is

$$[\mathbf{r} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - 1] + \left(\frac{-2}{3}\right)[\mathbf{r} \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + 2] = 0$$

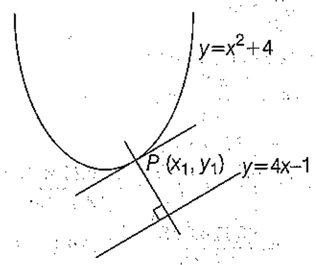
or $\quad 3[\mathbf{r} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - 1] - 2[\mathbf{r} \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + 2] = 0$

or $\quad \mathbf{r} \cdot (\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = 7$

4. *(d)* Given, curve $y = x^2 + 4$

and, line $y = 4x - 1$

Here, $y = x^2 + 4$



$$\therefore \quad \frac{dy}{dx} = 2x \quad \text{...(i)}$$

and $y = 4x - 1$

$$\frac{dy}{dx} = 4 \quad \text{... (ii)}$$

Let the required point be $P(x_1, y_1)$.

$$\therefore \quad \left.\frac{dy}{dx}\right|_P = 2x_1 \quad \text{... (iii)}$$

$\because$ Slopes will be equal.

$\therefore \quad 2x_1 = 4$ [from Eqs. (ii) and (iii)]

$\Rightarrow \quad x_1 = \frac{4}{2} = 2$

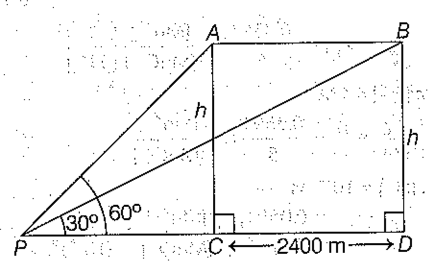
Now, the given point $P(x_1, y_1)$ lies on curve $y = x^2 + 4$,

$\therefore \quad y_1 = x_1^2 + 4$

$\Rightarrow \quad y_1 = 2^2 + 4 = 8$

Hence, required coordinate of $P = (2, 8)$

5. *(d)* Given, angle of elevation are 60° and then 30°.



Also, in 20 sec plane covers the distance from A to B with speed 432 km/h.

$\because \quad 432 \times \frac{5}{18}$ m/sec = 120 m/sec

$\therefore \quad AB = $ distance $= S \times T = 120 \times 20$

$= 2400$ m

In ΔPBD

$$\tan 30^\circ = \frac{BD}{PD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{PD}$$

$$\Rightarrow \quad PD = h\sqrt{3}$$

In ΔPAC

$$\tan 60^\circ = \frac{AC}{PC} \Rightarrow \sqrt{3} = \frac{h}{PC}$$

$$\Rightarrow \quad PC = h/\sqrt{3}$$

Now, $CD = PD - PC$

$$2400 = h\sqrt{3} - \frac{h}{\sqrt{3}} \qquad [\because AB = CD]$$

$$\Rightarrow \quad 2400 = h\left(\frac{3-1}{\sqrt{3}}\right)$$

$$\therefore \quad h = \frac{2400 \times \sqrt{3}}{2} = 1200 \times \sqrt{3}$$

$\therefore$ Required height $= 1200\sqrt{3}$ m

6. *(b)* Given, $n \geq 2$

Let $S = {}^2C_2 + {}^3C_2 + \ldots + {}^nC_2 = {}^{n+1}C_3$

Now, ${}^{n+1}C_2 + 2 \times ({}^2C_2 + {}^3C_2 + \ldots + {}^nC_2)$

$$= {}^{n+1}C_2 + 2 \times {}^{n+1}C_3$$

$$= ({}^{n+1}C_2 + {}^{n+1}C_3) + {}^{n+1}C_3$$

$$= {}^{n+2}C_3 + {}^{n+1}C_3 = \frac{(n+2)!}{3!(n-1)!} + \frac{(n+1)!}{3!(n-2)!}$$

$$= \frac{(n+2)(n+1)n(n-1)!}{3 \times 2 \times 1 \times (n-1)!} + \frac{(n+1) \times n \times (n-1) \times (n-2)!}{3 \times 2 \times 1 \times (n-2)!}$$

$$= \frac{n(n+1)}{6}[n+2+n-1]$$

$$= \frac{n(n+1)(2n+1)}{6}$$

7. *(d)* Given,

$$f(x) = \begin{cases} -55x & , \quad x < -5 \\ 2x^3 - 3x^2 - 120x & , \quad -5 \leq x < 4 \\ 2x^3 - 3x^2 - 36x + 10 & , \quad x \geq 4 \end{cases}$$

$$\therefore \quad f'(x) = \begin{cases} -55 & , \quad x < -5 \\ 6(x^2 - x - 20) & , \quad -5 \leq x < 4 \\ 6(x^2 - x - 6) & , \quad x \geq 4 \end{cases}$$

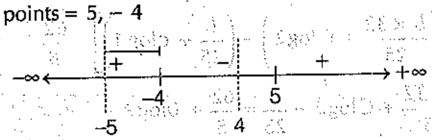
$$f'(x) = \begin{cases} -55 & , \quad x < -5 \\ 6(x-5)(x+4) & , \quad -5 \leq x < 4 \\ 6(x-3)(x+2) & , \quad x \geq 4 \end{cases}$$

For f to be increasing, $f'(x) > 0$

Now, $f'(x) = -55$ is always less than zero.

$f'(x) = 6(x-5)(x+4) < 0, -5 \leq x < 4$

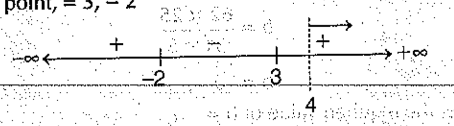
Critical points $= 5, -4$



$[x \in (-5, -4)$ ...(i)

and $f'(x) = 6(x-3)(x+2) < 0, x \geq 4$

Critical point, $= 3, -2$



$x \in (4, \infty)$ ... (ii)

From Eqs. (i) and (ii), $f(x)$ is increasing in $x \in (-5, -4) \cup (4, \infty)$

8. *(b)* Given, $f(0) = 1$,

$$f'(0) = 2,$$

$$f''(x) \neq 0$$

$$\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$$

$$\Rightarrow \quad f(x)f''(x) - f'(x)f'(x) = 0$$

$$\Rightarrow \quad \frac{f''(x)}{f'(x)} = \frac{f'(x)}{f(x)}$$

$$\Rightarrow \quad \int \frac{f''(x)}{f'(x)} dx = \int \frac{f'(x)}{f(x)} dx$$

$$\Rightarrow \quad \log f'(x) = \log f(x) + \log c \quad \ldots \text{(i)}$$

or $\quad f'(x) = cf(x)$

Now, put $x = 0$, we get

$$f'(0) = cf(0)$$

$$\Rightarrow \quad 2 = c \times 1$$

$$\Rightarrow \quad c = 2$$

Putting the value of $c = 2$ in Eq. (i), we get

$$\log f'(x) = \log f(x) + \log 2$$

$$\Rightarrow \quad f'(x) = 2f(x) \Rightarrow \int \frac{f'(x)}{f(x)} dx = \int 2dx$$

$$\Rightarrow \quad \log f(x) = 2x + D \Rightarrow f(x) = e^{2x+D}$$

$$\Rightarrow \quad f(x) = e^D \cdot e^{2x}$$

$$\Rightarrow \quad f(x) = k \cdot e^{2x} \qquad [\text{Let } k = e^D]$$

Put $x = 0$, we get

$$f(0) = k.e^0$$

$$\Rightarrow \quad 1 = k \Rightarrow f(x) = k \cdot e^{2x}$$

$$\therefore \quad f(x) = e^{2x}$$

Put $x = 1$, we get

$$f(1) = e^2$$

Clearly, e^2 lies in (6, 9).

9. *(d)* Given, line $x + \sqrt{3}y = 2\sqrt{3}$

and point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$

From options, we take the conic $x^2 + 9y^2 = 9$

Equation of any tangent at (x_1, y_1) is

$$xx_1 + 9 \cdot yy_1 = 9$$

$$\Rightarrow \quad \frac{3\sqrt{3}}{2}x + 9 \times \frac{1}{2} \times y = 9 \qquad \left[\because (x_1, y_1) = \left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)\right]$$

$$\Rightarrow \quad \frac{3\sqrt{3}}{2}(x + \sqrt{3}y) = 9$$

$$\Rightarrow \quad x + \sqrt{3}y = \frac{9 \times 2}{3\sqrt{3}}$$

$$\Rightarrow \quad x + \sqrt{3}y = 2\sqrt{3}$$

Clearly, $x + \sqrt{3}y = 2\sqrt{3}$ is a tangent to the curve $x^2 + 9y^2 = 9$.

10. *(b)* Let $I = \int_1^3 [x^2 - 2x - 2] dx$

$$= \int_1^3 [x^2 - 2x + 1 - 3] dx = \int_1^3 (x-1)^2 - 3] dx$$

$$= \int_1^3 [(x-1)^2] dx + \int_1^3 -3dx$$

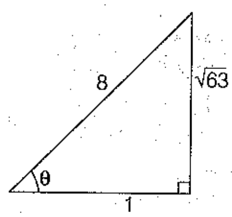
Put $x - 1 = t$; $dx = dt$, when $x = 1, t = 0$ and $x = 3, t = 2$

$$\therefore \quad I = -3[x]_1^3 + \int_0^2 [t^2] dt$$

$$= -6 + \int_0^1 0\,dt + \int_1^{\sqrt{2}} 1dt + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dt + \int_{\sqrt{3}}^{2} 3dt$$
$$= -6 + (0) + (\sqrt{2} - 1) + 2(\sqrt{3} - \sqrt{2}) + 3(2 - \sqrt{3})$$
$$= -6 + \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$$
$$I = -1 - \sqrt{2} - \sqrt{3}$$

11. *(a)* Given, $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$

Let $\sin^{-1}\frac{\sqrt{63}}{8} = \theta$



$\Rightarrow \quad \sin\theta = \frac{\sqrt{63}}{8} \Rightarrow \cos\theta = \frac{1}{8}$

Also, $\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$

$$= \sqrt{\frac{1+\frac{1}{8}}{2}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$$

$\therefore \tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right) = \tan\left(\frac{\theta}{4}\right)$

$$= \sqrt{\frac{1-\cos\frac{\theta}{2}}{1+\cos\frac{\theta}{2}}} = \sqrt{\frac{1-\frac{3}{4}}{1+\frac{3}{4}}} = \frac{1}{\sqrt{7}}$$

12. *(b)* Given, statement: $\sim p \wedge (p \vee q)$

Negative of given statement

$\sim[\sim p \wedge (p \vee q)]$

$= p \vee \sim [p \vee q)$ [by De Morgan's law]

$= p \vee (\sim p \wedge \sim q)$ [by De Morgan's law]

$= (p \vee \sim p) \wedge (p \vee \sim q)$ [Using distributive property]

$= t \wedge (p \vee \sim q)$

$= p \vee \sim q$

13. *(c)* Given, curve $\Rightarrow y = ax^2 + bx + c, x \in R$ and point (1, 2)

$\because$ The given curve passes through (1, 2).

$\therefore \quad 2 = a + b + c$

Also, slope of tangent of $y = ax^2 + bx + c$ is $\frac{dy}{dx} = 2ax + b$

$\because$ Tangent passes through origin (0, 0).

$\therefore \quad \left.\frac{dy}{dx}\right|_{(0,0)} = 2a \times 0 + b = b$ ... (i)

According to the question, tangent at origin is $y = x$

$\therefore$ Its slope is 1. ... (ii)

From Eqs. (i) and (ii),

$b = 1$

Also, $a + b + c = 2$

$\Rightarrow \quad a + c + 1 = 2 \Rightarrow a + c = 1$

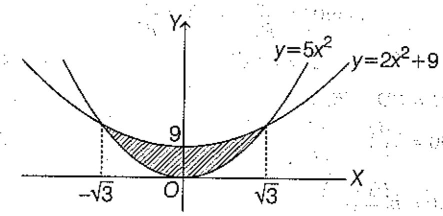
From the option look for $b = 1$ and $a + c = 1$

The only correct order triplet is $a = 1, b = 1, c = 0$.

14. *(b)* Given, $R = \{(x, y) : 5x^2 \le y \le 2x^2 + 9\}$

Here, we have two curves $y = 5x^2$ and $y = 2x^2 + 9$, point of intersection of both curves is find by solving both equations i.e.

$$5x^2 = 2x^2 + 9$$
$$\Rightarrow \quad x^2 = 3 \Rightarrow x = \pm\sqrt{3}$$



$\therefore \quad \text{Area} = \int_{-\sqrt{3}}^{\sqrt{3}} (2x^2 + 9 - 5x^2)\,dx$

$$= 2\int_0^{\sqrt{3}} (9 - 3x^2)\,dx$$
$$= 2[9x - x^3]_0^{\sqrt{3}}$$
$$= 2[9\sqrt{3} - 3\sqrt{3}]$$
$$= 12\sqrt{3} \text{ sq units}$$

15. *(b)* Given, curve $y = f(x)$ passes through (1, 2) and satisfies

$$x\frac{dy}{dx} + y = bx^4$$
$$\Rightarrow \quad x\frac{dy}{dx} + y = bx^4$$
$$\Rightarrow \quad \frac{dy}{dx} + \frac{y}{x} = bx^3$$
$$\text{IF} = e^{\int \frac{1}{x}\cdot dx} = x$$
$$\therefore \quad yx = \int bx^4\,dx = \frac{bx^5}{5} + C$$
$$\Rightarrow \quad y = \frac{bx^4}{5} + \frac{C}{x} = f(x) \quad \text{... (i)}$$

$\because$ This curve passes through (1, 2).

$$\therefore \quad 2 \times 1 = \frac{b \times (1)^5}{5} + C$$
$$\Rightarrow \quad 2 = \frac{b}{5} + C \quad \text{... (ii)}$$

Also, $\int_1^2 f(x)\,dx = \frac{62}{5}$

$$\Rightarrow \quad \int_1^2 \left(\frac{bx^4}{5} + \frac{C}{x}\right)dx = \frac{62}{5} \quad \text{[from Eq. (i)]}$$
$$\Rightarrow \quad \left[b \times \frac{x^5}{25} + C\log x\right]_1^2 = \frac{62}{5}$$
$$\Rightarrow \quad \left[\left(\frac{b \times 32}{25} + C\log 2\right) - \left(\frac{b}{25} + c\log 1\right)\right] = \frac{62}{5}$$
$$\Rightarrow \quad \frac{b \times 32}{25} + C\log 2 - \frac{b}{25} = \frac{62}{5} + 0\log 2$$

On comparing, we get

$$\frac{b}{25} \times 31 = \frac{62}{5} \text{ and } c = 0$$
$$b = \frac{62 \times 25}{31 \times 5}$$
$$b = 10$$

Hence, the required value of $b = 10$.

16. *(b)* Given, $f(0)=1$ ... (i)

$f(2)=e^2$ ... (ii)

$f'(x)=f'(2-x)$

Integrating w.r.t. x,

$f(x)=-f(2-x)+C$

Put $x=0$

$f(0)=-f(2)+C$

$\Rightarrow 1=-e^2+C$ [from Eqs. (i) and (ii)]

$\Rightarrow C=1+e^2$

$\therefore f(x)=-f(2-x)+1+e^2$

or $f(x)+f(2-x)=1+e^2$ ... (iii)

Let $I=\int_0^2 f(x)\,dx$ ... (iv)

Also, $I=\int_0^2 f(2-x)\,dx$ ... (v)

Now, adding Eqs. (iv) and (v),

$2I=\int_0^2[f(x)+f(2-x)]\,dx$

$2I=\int_0^2(1+e^2)\,dx$ [from Eq. (iii)]

$2I=2(1+e^2)$

$\therefore I=(1+e^2)$

17. *(c)* Given, A be a 3×3 matrix.

A is symmetric and B is skew-symmetric.

$\therefore A^T=A, B^T=-B$

Let $A^2B^2-B^2A^2=P$

$P^T=(A^2B^2-B^2A^2)^T=(A^2B^2)^T-(B^2A^2)^T$

$=(B^2)^T(A^2)^T-(A^2)^T(B^2)^T$

$=B^2A^2-A^2B^2=-(A^2B^2-B^2A^2)=-P$

$\Rightarrow P^T=-P$

P is skew-symmetric.

$\therefore |P|=0$

Hence, $PX=0$ have infinite solutions.

18. *(d)* Given, a, b and c are in AP.

(a, c), $(2, b)$ and (a, b) are vertices of triangle.

Centroid $=\left(\frac{10}{3},\frac{7}{3}\right)$

α and β are the roots of equation $ax^2+bx+1=0$

$\because a, b, c$ are in AP.

$\therefore 2b=a+c$ ... (i)

Centroid $=\left(\frac{a+2+a}{3},\frac{c+b+b}{3}\right)$

$=\left(\frac{2a+2}{3},\frac{c+2b}{3}\right)=\left(\frac{10}{3},\frac{7}{3}\right)$

$\Rightarrow \frac{2a+2}{3}=\frac{10}{3}$ and $\frac{c+2b}{3}=\frac{7}{3}$

$\Rightarrow a=4$

and $c+a+c=7$ [$\because 2b=a+c$]

$\Rightarrow 2c=7-4$ [$\because a=4$]

$c=3/2$

Also, $2b=a+c=4+\frac{3}{2}$

$\Rightarrow b=11/4$

Now, α and β are roots of $ax^2+bx+1=0$

$\therefore \alpha+\beta=\frac{-b}{a}=\frac{-11/4}{4}$

$\Rightarrow \alpha+\beta=\frac{-11}{16}\Rightarrow \alpha\beta=\frac{1}{a}=\frac{1}{4}$

$\Rightarrow \alpha\beta=\frac{1}{4}$

Now, $\alpha^2+\beta^2-\alpha\beta$

$=(\alpha+\beta)^2-3\alpha\beta=\left(\frac{-11}{16}\right)^2-3\times\frac{1}{4}$

$=\frac{121-192}{256}=\frac{-71}{256}$

19. *(d)* Given, $x-2y+0z=1$

$x-y+kx=-2$

$0x+ky+4z=6$

Here, $\Delta=\begin{vmatrix}1&-2&0\\1&-1&k\\0&k&4\end{vmatrix}=1(-4-k^2)+2(4)$

$=-4-k^2+8=4-k^2$

$\Delta_x=\begin{vmatrix}1&-2&0\\-2&-1&k\\6&k&4\end{vmatrix}=1(-4-k^2)+2(-8-6k)$

$=-4-k^2-16-12k=-k^2-12k-20$

If $\Delta\neq 0$, then it has unique solution i.e. $4-k^2\neq 0$

$\Rightarrow k\neq\pm 2$ for unique solution.

Also at $k=2$

$\Delta_x=-2^2-12\times 2-20=-48\neq 0$

Then, in this case it has no solution.

Hence, statement (A) and statement (D) both are correct.

20. *(c)* Given, set $=\{1, 2, 3, 4, 5\}$

Let the two subsets be A and B.

Then, $n(A\cap B)=2$ (as given in question)

$\therefore$ Required probability $=\frac{{}^5C_2\times 3^3}{4^5}=\frac{10\times 27}{2^{10}}=\frac{135}{2^9}$

21. (*) Given, $\binom{n}{r}=\begin{cases}{}^nC_r, & \text{if } n\geq r\geq 0\\ 0, & \text{otherwise}\end{cases}$

and $\sum_{i=0}^{k}\binom{10}{i}\binom{15}{k-i}+\sum_{i=0}^{k+1}\binom{12}{i}\binom{13}{k+1-i}$

$\because (1+x)^{10}={}^{10}C_0+{}^{10}C_1x+{}^{10}C_2x^2+\ldots+{}^{10}C_{10}x^{10}$

and $(1+x)^{15}={}^{15}C_0+{}^{15}C_1x+{}^{15}C_2x^2+\ldots+{}^{15}C_{15}x^{15}$

$\therefore \sum_{i=0}^{k}({}^{10}C_i)({}^{15}C_{k-i})={}^{10}C_0\cdot{}^{15}C_k+{}^{10}C_1\cdot{}^{15}C_{k-1}+\ldots+{}^{10}C_k\cdot{}^{15}C_0$

$\Rightarrow$ Coefficient of x^k in $(1+x)^{25}={}^{25}C_k$

Also, $\sum_{i=0}^{k+1}({}^{12}C_i)({}^{13}C_{k+1-i})={}^{12}C_0\cdot{}^{13}C_{k+1}$

$+{}^{12}C_1\cdot{}^{13}C_k+\ldots+{}^{12}C_{k+1}\cdot{}^{13}C_0$

$\Rightarrow$ Coefficient of x^{k+1} in $(1+x)^{25}={}^{25}C_{k+1}$

$\Rightarrow {}^{25}C_k+{}^{25}C_{k+1}={}^{26}C_{k+1}$

So, ${}^{26}C_{k+1}$ always exists.

Now k can be as larger as possible.

22. *(1)* Given, equation of line $\Rightarrow x-\lambda=2y-1=-2z$

$\Rightarrow \frac{x-\lambda}{1}=\frac{y-1/2}{\frac{1}{2}}=\frac{z}{-\frac{1}{2}}$

or $\frac{x-\lambda}{2}=\frac{y-1/2}{1}=\frac{z}{-1}$ ... (i)

Point on this line through which it passes is $\left(\lambda, \frac{1}{2}, 0\right)$.

Equation of another line $\Rightarrow x = y + 2\lambda = z - \lambda$

$$\Rightarrow \quad \frac{x}{1} = \frac{y - (-2\lambda)}{1} = \frac{z - \lambda}{1} \quad \text{... (ii)}$$

A point through which this line passes is $(0, -2\lambda, \lambda)$.

Now, distance between two skew lines

$$= \frac{|(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|}{|\mathbf{b}_1 \times \mathbf{b}_2|}$$

$$= \frac{\begin{vmatrix} \lambda & \frac{1}{2} + 2\lambda & -\lambda \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}} = \frac{\left|-5\lambda - \frac{3}{2}\right|}{\sqrt{14}}$$

According to the question, $\dfrac{\left|-5\lambda - \frac{3}{2}\right|}{\sqrt{14}} = \dfrac{\sqrt{7}}{2\sqrt{2}}$

$$\Rightarrow \quad |10\lambda + 3| = 7$$

$$\Rightarrow \quad 10\lambda + 3 = \pm 7$$

$$\Rightarrow \quad 10\lambda = 4, -10$$

$$\therefore \quad \lambda = \frac{2}{5} \text{ and } \lambda = -1$$

($\lambda = \frac{2}{5}$ is not possible as λ is an integer)

$$\therefore \quad \lambda = -1.$$

Hence, $|\lambda| = |-1| = 1$

23. *(2)* Given, $a + \alpha = 1$

$$b + \beta = 2$$

$$\because \quad a \cdot f(x) + \alpha \cdot f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x} \quad \text{... (i)}$$

Replace x by $\frac{1}{x}$,

$$af\left(\frac{1}{x}\right) + \alpha f(x) = \frac{b}{x} + \beta x \quad \text{... (ii)}$$

Adding Eqs. (i) and (ii), we get

$$(a + \alpha)\left[f(x) + f\left(\frac{1}{x}\right)\right] = \left(x + \frac{1}{x}\right)(b + \beta)$$

$$\Rightarrow \quad \frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} = \frac{b + \beta}{a + \alpha} = \frac{2}{1} = 2$$

24. *(56.25)* Let P be (h, k), $A(5, 0)$ and $B(-5, 0)$.

Given, $\quad PA = 3PB$

$$\Rightarrow \quad PA^2 = 9PB^2$$

$$\Rightarrow (h - 5)^2 + k^2 = 9[(h + 5)^2 + k^2]$$

$$\Rightarrow 8h^2 + 8k^2 + 100h + 200 = 0$$

$\therefore$ Locus of P is $x^2 + y^2 + \frac{25}{2}x + 25 = 0$

$$\text{Centre} = \left(\frac{-25}{4}, 0\right)$$

and $r^2 = \left(\frac{-25}{4}\right)^2 + 0^2 - 25$

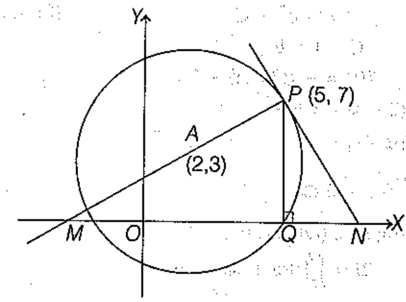
$$= \frac{625}{16} - 25 = \frac{225}{16}$$

$$\therefore \quad 4r^2 = 4 \times \frac{225}{16} = \frac{225}{4} = 56.25$$

25. *(1225)* Given, circle $(x - 2)^2 + (y - 3)^2 = 5^2$

$$c = (2, 3)$$

$$r = 5$$



Equation of normal at P (i.e. PA line)

$$\Rightarrow \quad (y - 7) = \left(\frac{7 - 3}{5 - 2}\right)(x - 5)$$

$$\Rightarrow \quad 3y - 21 = 4x - 20$$

$$\Rightarrow \quad 4x - 3y + 1 = 0$$

Therefore, $M = \left(\frac{-1}{4}, 0\right)$ [Put $y = 0$ in above equation]

Now, equation of tangent at P.

$$y - 7 = \frac{-3}{4}(x - 5) \quad \left[\because \text{slope of } PN = \frac{-1}{\text{Slope of } PA}\right]$$

$$\Rightarrow 4y - 28 = -3x + 15$$

$$\Rightarrow 3x + 4y = 43$$

Therefore, $N = \left(\frac{43}{3}, 0\right)$ [Put $y = 0$ in above equation]

$$\therefore \text{Area } (A) = \frac{1}{2} \times MN \times PQ$$

$$= \frac{1}{2} \times \left(\frac{43}{3} + \frac{1}{4}\right) \times 7$$

$$= \frac{1}{2} \times \frac{175}{12} \times 7$$

$$\therefore \quad 24A = 24 \times \frac{1}{2} \times \frac{175}{12} \times 7 = 1225$$

But this question is wrong as in question. It is mentioned that the triangle is formed with the positive X-axis which contradicts the solution.

26. *(11)* Given, 10 natural numbers $= 1, 1, 1, \ldots, 1, k$

According to the question, variance $(\sigma^2) < 10$

$$\because \quad \sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$$

$$\sigma^2 = \frac{(9 + k^2)}{10} - \left(\frac{9 + k}{10}\right)^2 < 10$$

$$\Rightarrow \quad 10(9 + k^2) - (81 + k^2 + 18k) < 1000$$

$$\Rightarrow \quad 90 + 10k^2 - 81 - k^2 - 18k < 1000$$

$$\Rightarrow \quad 9k^2 - 18k + 9 < 1000$$

$$\Rightarrow \quad (k - 1)^2 < \frac{1000}{9}$$

$$\Rightarrow \quad k - 1 < \frac{10\sqrt{10}}{3}$$

and $k-1>\frac{-10\sqrt{10}}{3} \Rightarrow k<\frac{10\sqrt{10}}{3}+1$

It is not possible because $k \in N$.

$\therefore$ Maximum possible integral value of k is 11.

27. *(3)* Let four numbers in GP be a, ar, ar^2, ar^3.

According to the question, $a + ar + ar^2 + ar^3 = \frac{65}{12}$ ...(i)

and $\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}+\frac{1}{ar^3}=\frac{65}{18}$

$\Rightarrow \frac{1}{a}\left(\frac{1+r+r^2+r^3}{r^3}\right)=\frac{65}{18}$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{a(1+r+r^2+r^3)}{\frac{1}{a}\frac{(1+r+r^2+r^3)}{r^3}}=\frac{65/12}{65/18}$$

$\Rightarrow a^2r^3=\frac{18}{12} \Rightarrow a^2r^3=\frac{3}{2}$

Also, product of first three terms = 1

$a \times ar \times ar^2 = 1$

$\Rightarrow a^3r^3 = 1$

$\Rightarrow a^3 \times \frac{3}{2a^2}=1$ $\left[\because r^3=\frac{3/2}{a^2}\right]$

$\Rightarrow a=\frac{2}{3}$

and $r^3=\frac{3/2}{(2/3)^2}=\left(\frac{3}{2}\right)^3 \Rightarrow r=\frac{3}{2}$

According to the question,

third term $=\alpha = ar^2=\frac{2}{3}\times\frac{3}{2}\times\frac{3}{2}=\frac{3}{2}$

$\therefore 2\alpha = 2\times\frac{3}{2}=3$

28. *(31650)* Given, total students = 10.

Number of groups = 3 (i.e. A, B and C)

Each group has atleast one student but group C has atmost 3 students.

$\therefore$ There are 3 cases depending on number of students in group C.

Case I C has 1 student, then $\left.\begin{matrix}A\\B\end{matrix}\right] \leftarrow$ 9 students.

$\therefore$ Number of ways $= {}^{10}C_1 \times [2^9-2]$

Case II C has 2 students, then $\left.\begin{matrix}A\\B\end{matrix}\right] \leftarrow$ 8 Students.

$\therefore$ Number of ways $= {}^{10}C_2 \times [2^8-2]$

Case III C has 3 students, then $\left.\begin{matrix}A\\B\end{matrix}\right] \leftarrow$ 7 Students.

$\therefore$ Number of ways $= {}^{10}C_3 \times [2^7-2]$

$\therefore$ Required number of possibilities

$= {}^{10}C_1(2^9-2)+{}^{10}C_2(2^8-2)+{}^{10}C_3(2^7-2)$

$=2^7[{}^{10}C_1\times 4+{}^{10}C_2\times 2+{}^{10}C_3]-20-90-240$

$=128[40+90+120]-350$

$=(128\times 250)-350=31650$

29. *(310)* Given, $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}}+\frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}}=k$

$\because -1+i\sqrt{3}=2e^{i2\pi/3}$

$1+i\sqrt{3}=2e^{i\pi/3}$

$1-i=\sqrt{2}e^{-i\pi/4}$

$1+i=\sqrt{2}e^{i\pi/4}$

Now, $\frac{\left(2e^{i\frac{2\pi}{3}}\right)^{21}}{(\sqrt{2}e^{-i\pi/4})^{24}}+\frac{(2e^{i\pi/3})^{21}}{(\sqrt{2}e^{i\pi/4})^{24}}$

$=\frac{2^{21}\cdot e^{i14\pi}}{2^{12}\cdot e^{-i6\pi}}+\frac{2^{21}\cdot e^{i7\pi}}{2^{12}\cdot e^{i6\pi}}$

$=2^9\cdot e^{i20\pi}+2^9\cdot e^{i\pi}$

$=2^9(1)+2^9(-1)$

$\Rightarrow 2^9-2^9=0=k$ (given)

$\therefore n=[|k|]=0$

Now, $\sum_{j=0}^{5}(j+5)^2-\sum_{j=0}^{5}(j+5)$ $[\because n=0]$

$=[5^2+6^2+7^2+8^2+9^2+10^2]-[5+6+7+8+9+10]$

$=[(1^2+2^2+3^2+...+10^2)-(1^2+2^2+...+4^2)]$
$-[(1+2+3+...+10)-(1+2+3+4)]$

$=\left[\frac{10\times11\times21}{6}-\frac{4\times5\times9}{6}\right]-\left[\frac{10\times11}{2}-\frac{4\times5}{2}\right]$

$=(385-30)-(55-10)$

$=355-45=310$

30. *(2)* Given, equation $(x+1)^2+|(x-5)|=\frac{27}{4}$

Case I For $x\ge 5$

$\Rightarrow (x+1)^2+(x-5)=\frac{27}{4}$

$\Rightarrow x^2+3x-4=\frac{27}{4}$

$\Rightarrow 4x^2+12x-43=0$

$\therefore x=\frac{-12\pm\sqrt{144+688}}{8}$

$=\frac{-12\pm\sqrt{832}}{8}$

$=\frac{-12\pm 28.8}{8}$

$x=\frac{-3\pm 7.2}{2}$

$x=\frac{-3+7.2}{2}, \frac{-3-7.2}{2}$

Both the values are less than 5.

$\therefore$ No solution from here.

Case II $x<5$

$\Rightarrow (x+1)^2-(x-5)=\frac{27}{4}$

$\Rightarrow x^2+x+6-\frac{27}{4}=0$

$\Rightarrow 4x^2+4x-3=0$

$\Rightarrow x=\frac{-4\pm\sqrt{16+48}}{8}$

$=\frac{-4\pm 8}{8}$

$\Rightarrow x=\frac{-12}{8},\frac{4}{8}$, both are less than 5.

$\therefore$ These values must be the solution.

Hence, here 2 real roots are possible.

JEE Main 2021

25 FEBRUARY SHIFT I

PHYSICS

Section A : Objective Type Questions

1. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion (A) When a rod lying freely is heated, no thermal stress is developed in it.

Reason (R) On heating, the length of the rod increases.

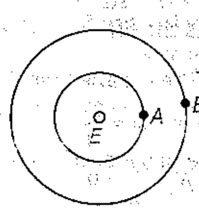
In the light of the above statements, choose the correct answer from the options given below

a. Both A and R are true but R is not the correct explanation of A.
b. A is false but R is true.
c. A is true but R is false.
d. Both A and R are true and R is the correct explanation of A.

2. A student is performing the experiment of resonance column. The diameter of the column tube is 6 cm. The frequency of the tuning fork is 504 Hz. Speed of the sound at the given temperature is 336 m/s. The zero of the meter scale coincides with the top end of the resonance column tube. The reading of the water level in the column when the first resonance occurs is

a. 13 cm b. 16.6 cm
c. 18.4 cm d. 14.8 cm

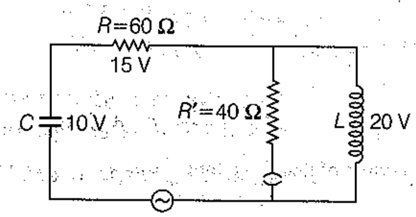
3. Two satellites *A* and *B* of masses 200 kg and 400 kg are revolving around the Earth at height of 600 km and 1600 km, respectively. If T_A and T_B are the time periods of *A* and *B* respectively, then the value of $T_B - T_A$ is



(Given, radius of Earth = 6400 km, mass of Earth $= 6 \times 10^{24}$ kg)

a. 1.33×10^3 s b. 3.33×10^2 s
c. 4.24×10^3 s d. 4.24×10^2 s

4. The angular frequency of alternating current in an *L-C-R* circuit is 100 rad/s. The components connected are shown in the figure. Find the value of inductance of the coil and capacity of condenser.



a. 0.8 H and 150 μF b. 0.8 H and 250 μF
c. 1.33 H and 250 μF d. 1.33 H and 150 μF

5. A proton, a deuteron and an α-particle are moving with same momentum in a uniform magnetic field. The ratio of magnetic forces acting on them is .... and their speed is .... in the ratio.

a. 1 : 2 : 4 and 2 : 1 : 1 b. 2 : 1 : 1 and 4 : 2 : 1
c. 4 : 2 : 1 and 2 : 1 : 1 d. 1 : 2 : 4 and 1 : 1 : 2

6. Given, below are two statements

Statement I A speech signal of 2 kHz is used to modulate a carrier signal of 1 MHz. The bandwidth requirement for the signal is 4 kHz.

Statement II The side band frequencies are 1002 kHz and 998 kHz. In the light of the above statements, choose the correct answer from the options given below

a. Statement I is true but Statement II is false.
b. Statement I is false but Statement II is true.
c. Both Statement I and Statement II are true.
d. Both Statement I and Statement II are false.

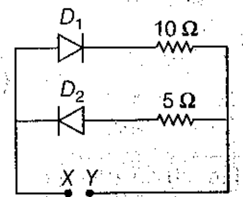
7. If the time period of a 2 m long simple pendulum is 2 s, the acceleration due to gravity at the place, where pendulum is executing SHM is

a. $\pi^2 \text{ms}^{-2}$ b. 9.8 ms^{-2}
c. $2\pi^2 \text{ ms}^{-2}$ d. 16 ms^{-2}

8. The pitch of the screw gauge is 1 mm and there are 100 divisions on the circular scale. When nothing is put in between the jaws, the zero of the circular scale lies 8 divisions below the reference line. When a wire is placed between the jaws, the first linear scale division is clearly visible while 72nd division on circular scale coincides with the reference line. The radius of the wire is

a. 1.64 mm b. 0.82 mm c. 1.80 mm d. 0.90 mm

9. A 5 V battery is connected across the points X and Y. Assume D_1 and D_2 to be normal silicon diodes. Find the current supplied by the battery, if the positive terminal of the battery is connected to point X.



a. ~ 0.5 A **b.** ~ 1.5 A
c. ~ 0.86 A **d.** ~ 0.43 A

10. An α-particle and a proton are accelerated from rest by a potential difference of 200 V. After this, their de-Broglie wavelengths are λ_α and λ_p, respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$ is

a. 3.8 **b.** 8 **c.** 7.8 **d.** 2.8

11. A diatomic gas having $C_p = \frac{7}{2}R$ and $C_V = \frac{5}{2}R$, is heated at constant pressure. The ratio $dU : dQ : dW$ is

a. 5 : 7 : 3 **b.** 5 : 7 : 2
c. 3 : 7 : 2 **d.** 3 : 5 : 2

12. An engine of a train moving with uniform acceleration, passes the signal-post with velocity u and the last compartment with velocity v. The velocity with which middle point of the train passes the signal post is

a. $\sqrt{\frac{v^2+u^2}{2}}$ **b.** $\frac{v-u}{2}$ **c.** $\frac{u+v}{2}$ **d.** $\sqrt{\frac{v^2-u^2}{2}}$

13. Match List-I with List-II

List-I	List-II
A. h (Planck's constant)	1. $[M L T^{-1}]$
B. E (kinetic energy)	2. $[M L^2 T^{-1}]$
C. V (electric potential)	3. $[M L^2 T^{-2}]$
D. P (linear momentum)	4. $[M L^2 I^{-1} T^{-3}]$

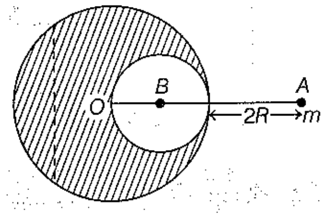
Choose the correct answer from the options given below.

	A	B	C	D		A	B	C	D
a.	3	4	2	1	**b.**	2	3	4	1
c.	1	2	4	3	**d.**	3	2	4	1

14. Magnetic fields at two points on the axis of a circular coil at a distance of 0.05 m and 0.2 m from the centre are in the ratio 8 : 1. The radius of coil is

a. 0.2 m **b.** 0.1 m
c. 0.15 m **d.** 1.0 m

15. A solid sphere of radius R gravitationally attracts a particle placed at $3R$ from its centre with a force F_1. Now, a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is

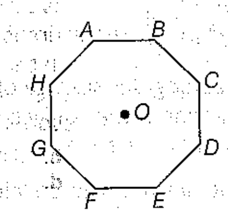


a. 25 : 36 **b.** 36 : 25
c. 50 : 41 **d.** 41 : 50

16. Two radioactive substances X and Y originally have N_1 and N_2 nuclei, respectively. Half-life of X is half of the half-life of Y. After three half-lives of Y, number of nuclei of both are equal. The ratio $\frac{N_1}{N_2}$ will be equal to

a. $\frac{1}{8}$ **b.** $\frac{3}{1}$
c. $\frac{8}{1}$ **d.** $\frac{1}{3}$

17. In an octagon $ABCDEFGH$ of equal side, what is the sum of $\mathbf{AB} + \mathbf{AC} + \mathbf{AD} + \mathbf{AE} + \mathbf{AF} + \mathbf{AG} + \mathbf{AH}$ if, $\vec{AO} = 2\hat{i} + 3\hat{j} - 4\hat{k}$?



a. $-16\hat{i} - 24\hat{j} + 32\hat{k}$ **b.** $16\hat{i} + 24\hat{j} - 32\hat{k}$
c. $16\hat{i} + 24\hat{j} + 32\hat{k}$ **d.** $16\hat{i} - 24\hat{j} + 32\hat{k}$

18. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

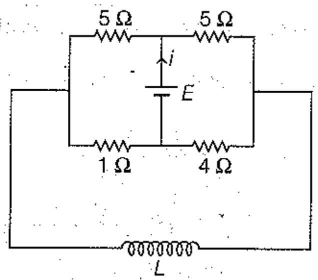
Assertion (A) The escape velocities of planet A and B are same. But A and B are of unequal mass.

Reason (R) The product of their mass and radius must be same, $M_1R_1 = M_2R_2$

In the light of the above statements, choose the most appropriate answer from the options given below.

a. Both A and R are correct but R is not the correct explanation of A.
b. A is correct but R is not correct.
c. Both A and R are correct and R is the correct explanation of A.
d. A is not correct but R is correct.

19. The current (i) at time $t = 0$ and $t = \infty$ respectively for the given circuit is



a. $\frac{18E}{55}, \frac{5E}{18}$ b. $\frac{10E}{33}, \frac{5E}{18}$ c. $\frac{5E}{18}, \frac{18E}{55}$ d. $\frac{5E}{18}, \frac{10E}{33}$

20. Two coherent light sources having intensity in the ratio $2x$ produce an interference pattern. The ratio $\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$ will be

a. $\frac{2\sqrt{2x}}{x+1}$ b. $\frac{\sqrt{2x}}{2x+1}$ c. $\frac{\sqrt{2x}}{x+1}$ d. $\frac{2\sqrt{2x}}{2x+1}$

Section B : Numerical Type Questions

21. A transmitting station releases waves of wavelength 960 m. A capacitor of 2.56 μF is used in the resonant circuit. The self-inductance of coil necessary for resonance is .... $\times 10^{-8}$ H.

22. The electric field in a region is given

$\mathbf{E} = \left(\frac{3}{5}E_0\hat{\mathbf{i}} + \frac{4}{5}E_0\hat{\mathbf{j}}\right)\frac{\text{N}}{\text{C}}$. The ratio of flux of reported field through the rectangular surface of area 0.2 m^2 (parallel to YZ-plane) to that of the surface of area 0.3 m^2 (parallel to XZ- plane) is $a : b$, where a = ...

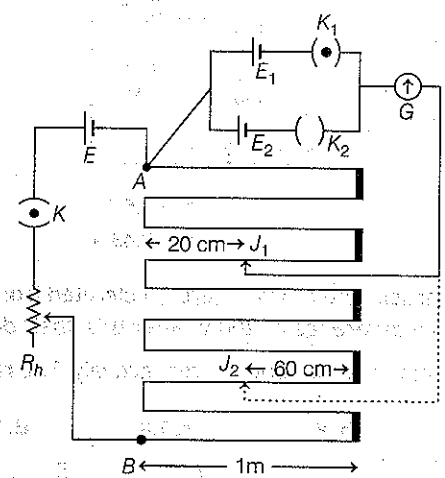
[Here $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are unit vectors along X, Y and Z-axes, respectively]

23. In a certain thermodynamical process, the pressure of a gas depends on its volume as kV^3. The work done when the temperature changes from 100°C to 300°C will be .... nR, where n denotes number of moles of a gas.

24. A small bob tied at one end of a thin string of length 1m is describing a vertical circle, so that the maximum and minimum tension in the string are in the ratio 5 : 1. The velocity of the bob at the highest position is .... m/s.

(Take, $g = 10$ m/s^2)

25. In the given circuit of potentiometer, the potential difference E across AB (10 m length) is larger than E_1 and E_2 as well. For key K_1(closed), the jockey is adjusted to touch the wire at point J_1, so that there is no deflection in the galvanometer. Now, the first battery (E_1) is replaced by second battery (E_2) for working by making K_1 open and K_2 closed. The galvanometer gives then null deflection at J_2. The value of $\frac{E_1}{E_2}$ is $\frac{a}{b}$, where a = ...



26. The same size images are formed by a convex lens when the object is placed at 20 cm or at 10 cm from the lens. The focal length of convex lens is .... cm.

27. 512 identical drops of mercury are charged to a potential of 2 V each. The drops are joined to form a single drop. The potential of this drop is .... V.

28. A coil of inductance 2H having negligible resistance is connected to a source of supply whose voltage is given by $V = 3t$ V (where, t is in second). If the voltage is applied when $t = 0$, then the energy stored in the coil after 4 s is .... J.

29. A monoatomic gas of mass 4.0 u is kept in an insulated container. Container is moving with velocity 30 m/s. If container is suddenly stopped, then change in temperature of the gas (R = gas constant) is $\frac{x}{3R}$. Value of x is ....

30. The potential energy (U) of a diatomic molecule is a function dependent on r (interatomic distance) as $U = \frac{\alpha}{r^{10}} - \frac{\beta}{r^5} - 3$

where, α and β are positive constants. The equilibrium distance between two atoms will be $\left(\frac{2\alpha}{\beta}\right)^{\frac{a}{b}}$, where a = ...